In the given circuit,

\[ \nu(t) = 56 \ e^{-2\times10^3 t} \ V, \ t > 0 \]

\[ i(t) = 7 \ e^{-2\times10^3 t} \ mA, \ t > 0 \]

References

Section Break  Difficulty: Easy  Learning Objective: Understand solutions to unforced, first order linear differential equations.
1. Award: **10.00 points**

Calculate the time constant \( \tau \).

The time constant \( \tau \) is \( 3.57 \pm 2\% \) ms.

**Explanation:**

The time constant is given by \( \tau = \frac{1}{RC} = 3.57 \text{ ms} \)

The time constant \( \tau \) is 3.57 ms.

**Hints**

**Hint #1**

**References**

**Worksheet** Difficulty: Easy

Learning Objective: Understand solutions to unforced, first order linear differential equations.
2. **Award: 10.00 points**

Find the time constant for the RC circuit in the given figure. Assume $R = 12 \, \Omega$.

The time constant for the RC circuit in the given figure is $3.00 \, \pm \, 2\%$ s.

**Explanation:**

The time constant is given by $\tau = R_{th}C$, where $R_{th}$ is the Thévenin equivalent at the capacitor terminals.

The value of $R_{th}$ can be determined as follows:

$$R_{th} = \frac{120}{80 + 12} \Omega = \left( \frac{120 \Omega \times 80 \Omega}{120 \Omega + 80 \Omega} \right) + 12 \, \Omega = 60.00 \, \Omega$$

Therefore, $\tau = 60.00 \, \Omega \times 0.05 \, F = 3.00 \, s$

The time constant for the RC circuit in the given figure is 3.00 s.
3. **Award: 10.00 points**

The switch in the given figure has been in position A for a long time. Assume the switch moves instantaneously from A to B at \( t = 0 \). Find \( v \) for \( t > 0 \). Assume \( R = 2 \text{ k}\Omega \).

The voltage \( v(t) = v(0) e^{-t/\tau} \), where \( v(0) = 40.0 \pm 2\% \text{ V} \) and \( \tau = 0.02 \pm 2\% \text{ s} \).

**Explanation:**

For \( t < 0 \), \( v(0) = 40 \text{ V} \)

For \( t > 0 \), we have a source-free RC circuit.

\[
\tau = RC = 2 \text{ k}\Omega \times 10 \mu\text{F} = 0.02 \text{ s}
\]

\[
v(t) = v(0) e^{-t/\tau} = 40 e^{-50t} \text{ V}
\]

The voltage \( v(t) = v(0) e^{-t/\tau} \), where \( v(0) = 40.0 \text{ V} \) and \( \tau = 0.02 \text{ s} \).

**Hints**

**Hint #1**

**References**

**Worksheet** Difficulty: Medium  
Learning Objective: Understand solutions to unforced, first order linear differential equations.
4. **Award: 10.00 points**

In the given circuit, find the unknown quantities of \( i(t) \) for \( t > 0 \) if \( i(0) = 5 \) A. Assume \( L = 8 \) H.

\[ i(t) = 5e^{-t/\tau} \text{ A}, \quad \text{where} \quad \tau = 0.27 \pm 2\% \text{ s}. \]

**Explanation:**

The given circuit is as shown below:

To find \( R_{TH} \), we replace the inductor by a 1-V voltage source as shown above.

\[ 10i_1 - 1 + 40i_2 = 0 \]

But \( i = i_2 + \frac{i}{2} \) and \( i = i_1 \)

i.e., \( i_1 = 2i_2 = i \)

\[ 10i - 1 + 20i = 0 \rightarrow i = \frac{1}{30} \text{ A} \]

The Thévenin resistance is given as

\[ R_{TH} = \frac{1}{\tau} = 30 \text{ \Omega} \]

The time constant is given by
\[ \tau = \frac{L}{R_{TH}} = \frac{V}{2 \pi \omega} = 0.27 \text{ s} \]

Therefore, the current \( i(t) = 5 e^{-\frac{t}{\tau}} \text{ A} \)

The current \( i(t) = 5 e^{-t/\tau} \text{ A} \), where \( \tau = 0.27 \text{ s} \).

**Hints**

**Hint #1**

**References**

**Worksheet** Difficulty: Medium Learning Objective: Understand solutions to unforced, first order linear differential equations.
In the given circuit, find the value of $R$ for which the steady-state energy stored in the inductor will be 1.1 J.

The value of $R$ is 11.472 ± 1% Ω.

Explanation:

The circuit can be replaced by its Thévenin equivalent shown below.

\[
V_{TH} = \frac{80\Omega}{80\Omega + 40\Omega} (60\text{ V}) = 40\text{ V}
\]

\[
R_{TH} = 40\ \Omega \parallel 80\ \Omega + R = \frac{80\Omega}{8\Omega + R}
\]

\[
I = i(0) = i(\infty) = \frac{V_{TH}}{R_{TH}} = \frac{40\text{ V}}{8\Omega} = 5\text{ A}
\]

The value of $R$ can be calculated as follows:

\[
\omega = \frac{1}{2}LI^2 = \left(0.5\right) \times \left(2\right) \times \left(\frac{40\text{ V}}{8\Omega + R}\right)^2 = 1.10 \ \text{J}
\]

\[
\Rightarrow \frac{40\text{ V}}{R + \frac{80\Omega}{8\Omega}} = 1.05
\]

\[
\Rightarrow R + \left(\frac{80\Omega}{8\Omega}\right) = 38.14
\]

\[
\Rightarrow R = 38.14 - \frac{80\Omega}{8\Omega}
\]

\[
\Rightarrow R = 11.472\ \Omega
\]

The value of $R$ is 11.472 Ω.
References

Worksheet | Difficulty: Medium | Learning Objective: Understand solutions to unforced, first order linear differential equations.
Express \( v(t) \) in the given figure in terms of step functions.

\[
\begin{align*}
\text{Hint #1} \\
\text{References}
\end{align*}
\]

- \( v(t) = (5u(t+1) + 10(t-1) - 25u(t) + 15u(t-2)) \text{ V} \)
- \( v(t) = (5u(t-1) + 10u(t) - 25u(t+2) + 15u(t+2)) \text{ V} \)
- \( v(t) = (5u(t+1) + 10u(t) - 25u(t-1) + 15u(t-2)) \text{ V} \)
- \( v(t) = (5u(t-2) + 10u(t-1) - 25u(t) + 15u(t+1)) \text{ V} \)

Learning Objective: Understand singularity equations and their importance in solving linear differential equations.
7. Award: **10.00 points**

The voltage across a 10-mH inductor is \(42 \delta(t - 2)\) mV. Find the inductor current, assuming that the inductor is initially uncharged.

The inductor current is \(i(t) = 4.2 \pm 2\% \, u(t - 2)\) A.

**Explanation:**

The inductor current can be determined as follows:

\[
i(t) = \frac{1}{L} \int_0^t v(\tau) \, d\tau + i(0)
\]

\[
i(t) = \frac{10^{-3}}{10 \times 10^{-3}} \int_0^t 42 \delta(\tau - 2) \, d\tau + 0 = 4.2 u(t - 2) \, \text{A}
\]

It should be noted that the integration of the impulse function \(\delta(t - t_o)\) produces the unit step \(u(t - t_o)\). Whatever the multiplier, \((f(t))\) of the impulse function at \(t = t_o\) ends up multiplying the unit step by the same amount \((f(t_o))\). In this case \(f(2) = 4.2\).

\[
i(t) = 4.2 \, u(t - 2) \, \text{A}
\]

The inductor current is \(4.2 \, u(t - 2)\) A.
8. Award: **10.00 points**

Find the solution of the differential equation \( \frac{dv}{dt} + 6v = 0, \ v(0) = -1 \) V.

The solution of the given differential equation is \(- (e^{-\left(\frac{6}{\pm 2\%}\right)}t) \) V.

Explanation:

The given differential equation can be solved as follows:
\[
v = A \, e^{-6t}, \quad v(0) = A = -1
\]

Therefore, \( v(t) = -e^{-6t} \) V.

The solution of the given differential equation is \(- e^{-6t} \) V.
9. **Award: 10.00 points**

Identify the solution of the following differential equation, subject to the stated initial condition.

\[
\frac{dv}{dt} = -3 (1 + e^{t/2}) v, \quad t < 0
\]

\[
v(t) = -3 (1 + e^{t/2}) u(t) V, \quad t > 0
\]

\[
v(t) = 3 (1 - e^{t/2}) u(t) V, \quad t > 0
\]

\[
v(t) = 3 (1 - e^{t/2}) V, \quad t < 0
\]

The given differential equation can be solved as below:

\[
v(t) = A + B e^{t/2}, \quad t > 0
\]

\[
A = -3, \quad v(0) = -6 = -3 + B \quad \rightarrow \quad B = -3
\]

\[
v(t) = -3 (1 + e^{t/2}) u(t) V, \quad t > 0
\]

The solution for the given differential equation is \( v(t) = -3 (1 + e^{t/2}) u(t) V, \quad t > 0 \).
A circuit is described by

\[ 2 \frac{dy}{dx} + y = 10. \]

References

Section Break Difficulty: Medium Learning Objective: Understand singularity equations and their importance in solving linear differential equations.
If \( v(0) = 3 \), find \( v(t) \) for \( t \geq 0 \).

The voltage \( v(t) = 10 \pm 2\% + (-7 \pm 2\%) \left( e^{-0.50 \pm 2\% t} \right) \times u(t) \) V.

**Explanation:**

Let \( v = v_h + v_p \) and \( v_p = 10 \).

\[
\dot{v}_h + \frac{1}{2} v_h = 0 \quad \Rightarrow \quad v_h = A e^{-\frac{t}{2}}
\]

\[
v = 10 + A e^{-0.50 t}
\]

\[
v(0) = 3 = 10 + A \quad \Rightarrow \quad A = -7
\]

\[
v = 10 + (-7) e^{-0.50 t} \times u(t) \) V.

The voltage \( v(t) = 10 + (-7) \left( e^{-0.50 t} \right) u(t) \) V.

**Hints**

- Hint #1
- Hint #2

**References**

**Worksheet** Difficulty: Medium

Learning Objective: Understand singularity equations and their importance in solving linear differential equations.