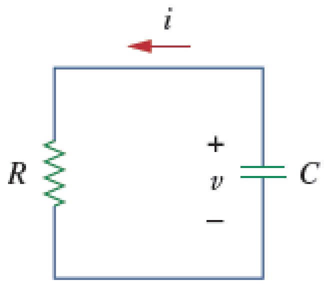
In the given circuit,
$v(t) = 56\ e^{-280\,t}$ V, $t > 0$

$i(t) = 7\ e^{-280\,t}$ mA, $t > 0$



References

Section Break Difficulty: Easy Learning Objective: Understand solutions to unforced, first order linear differential equations.

1. Award: **10.00 points**

Calculate the time constant τ.

The time constant τ is *3.57 ± 2%* ms.

Explanation:

The time constant is given by $\tau = RC = \frac{1}{280} = 3.57\ \mathrm{ms}$

The time constant τ is 3.57 ms.

Hints

Hint #1

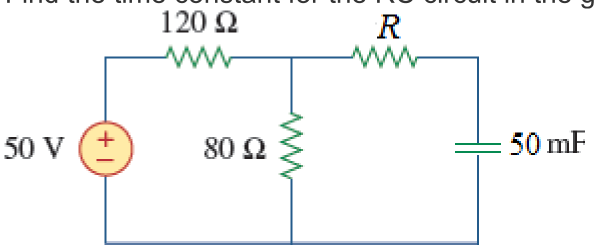
References

Worksheet Difficulty: Easy Learning Objective: Understand solutions to unforced, first order linear differential equations.

2.

Award: **10.00 points**

Find the time constant for the RC circuit in the given figure. Assume R = 12 Ω.



The time constant for the RC circuit in the given figure is *3.00 ± 2%* s.

Explanation:

The time constant is given by $\tau = R_{th}C$, where R_{th} is the Thévenin equivalent at the capacitor terminals.

The value of R_{th} can be determined as follows:

$$R_{th} = 120\ \Omega \,||\, 80\ \Omega + 12\ \Omega = \left(\frac{120\,\Omega \times 80\,\Omega}{120\,\Omega + 80\,\Omega}\right) + 12\ \Omega = 60.00\ \Omega$$

Therefore, $\tau = 60.00\ \Omega \times 0.05\ \text{F} = 3.00\ \text{s}$

The time constant for the RC circuit in the given figure is 3.00 s.

Hints

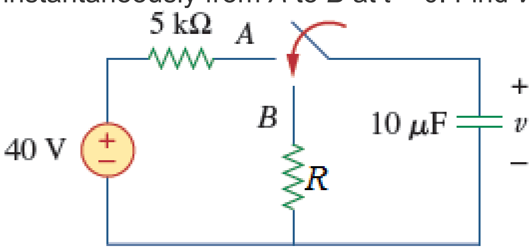
Hint #1

References

Worksheet Difficulty: Easy Learning Objective: Understand solutions to unforced, first order linear differential equations.

3. Award: **10.00 points**

The switch in the given figure has been in position *A* for a long time. Assume the switch moves instantaneously from *A* to *B* at $t = 0$. Find *v* for $t > 0$. Assume $R = 2$ kΩ.



The voltage $v(t) = v(0)\ e^{-t/\tau}$, where $v(0)$ = 40.0 ± 2% V and τ = 0.02 ± 2% s.

Explanation:

For $t < 0$, $v(0) = 40$ V

For $t > 0$, we have a source-free RC circuit.

$$\tau = RC = 2\ \text{k}\Omega \times 10\ \mu\text{F} = 0.02\ \text{s}$$

$$v(t) = v(0)\ e^{-t/\tau} = 40\ e^{-50.0t}\ \text{V}$$

The voltage $v(t) = v(0)\ e^{-t/\tau}$, where $v(0)$ = 40.0 V and τ = 0.02 s.

Hints

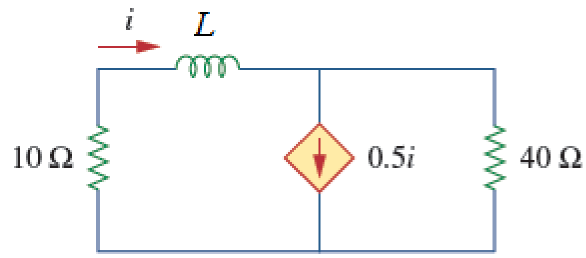
Hint #1

References

Worksheet | Difficulty: Medium | Learning Objective: Understand solutions to unforced, first order linear differential equations.

4. Award: **10.00 points**

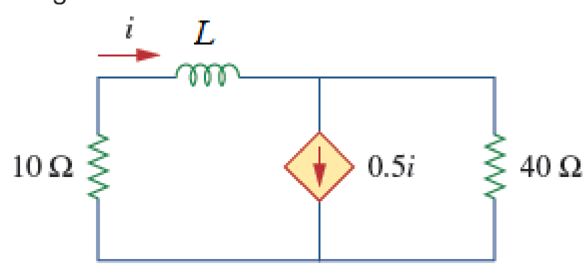
In the given circuit, find the unknown quantities of *i*(*t*) for *t* > 0 if *i*(0) = 5 A. Assume *L* = 8 H.

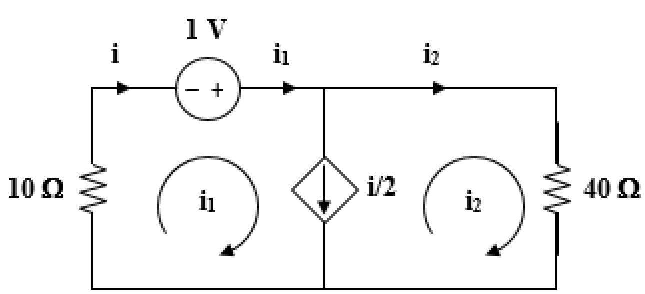


The current $i(t) = 5\,e^{-t/\tau}$ A, where τ = *0.27 ± 2%* s.

Explanation:

The given circuit is as shown below:





To find R_{TH}, we replace the inductor by a 1-V voltage source as shown above.

$10i_1 - 1 + 40\,i_2 = 0$

But $i = i_2 + \frac{i}{2}$ and $i = i_1$

i.e., $i_1 = 2i_2 = i$

$10i - 1 + 20i = 0 \rightarrow i = \frac{1}{30}$ A

The Thévenin resistance is given as

$R_{TH} = \frac{1}{i} = 30\ \Omega$

The time constant is given by

$\tau = \frac{L}{R_{TH}} = \frac{8 \text{ H}}{30 \, \Omega} = 0.27 \text{ s}$

Therefore, the current $i(t) = 5e^{-t/0.27} \text{A}$

The current $i(t) = 5e^{-t/\tau} \text{ A}$, where τ = 0.27 s.

Hints

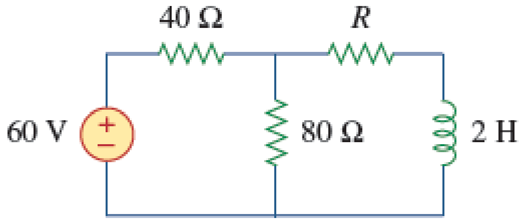
Hint #1

References

Worksheet Difficulty: Medium Learning Objective: Understand solutions to unforced, first order linear differential equations.

5. Award: **10.00 points**

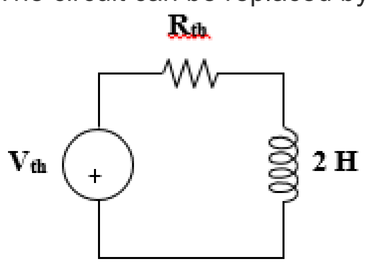
In the given circuit, find the value of *R* for which the steady-state energy stored in the inductor will be 1.1 J.



The value of *R* is *11.472 ± 1%* Ω.

Explanation:

The circuit can be replaced by its Thévenin equivalent shown below.



$$V_{TH} = \frac{80\ \Omega}{80\ \Omega + 40\ \Omega}(60\ \text{V}) = 40\ \text{V}$$

$$R_{TH} = 40\ \Omega \,||\, 80\ \Omega + R = \frac{80\ \Omega}{3\ \Omega} + R$$

$$I = i\left(0\right) = i\left(\infty\right) = \frac{V_{TH}}{R_{TH}} = \frac{40\ \text{V}}{\frac{80\ \Omega}{3\ \Omega} + R}$$

The value of *R* can be calculated as follows:

$$w = \frac{1}{2}LI^2 = \left(0.5\right) \times \left(2\right) \times \left(\frac{40\ \text{V}}{\frac{80\ \Omega}{3\ \Omega} + R}\right)^2 = 1.10\ \text{J}$$

$$\rightarrow \frac{40\ \text{V}}{R + \frac{80\ \Omega}{3\ \Omega}} = 1.05$$

$$\rightarrow R + \frac{80\ \Omega}{3\ \Omega} = 38.14$$

$$\rightarrow R = 38.14 - \frac{80\ \Omega}{3\ \Omega}$$

$$\rightarrow R = 11.472\ \Omega$$

The value of *R* is 11.472 Ω.

Hints

Hint #1

Hint #2

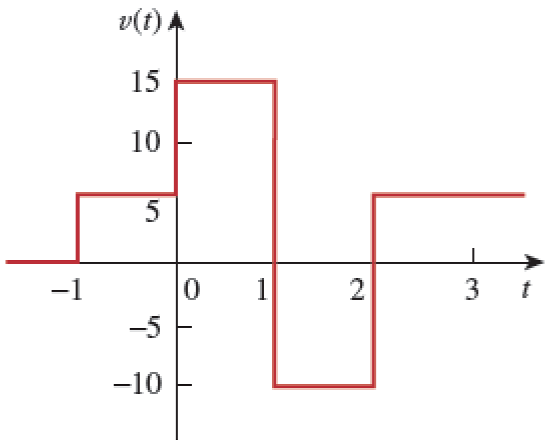
Hint #3

References

Worksheet	Difficulty: Medium	Learning Objective: Understand solutions to unforced, first order linear differential equations.

6. Award: **10.00 points**

Express $v(t)$ in the given figure in terms of step functions.



- $v(t) = (5u(t+1) + 10(t-1) - 25u(t) + 15u(t-2))$ V
- $v(t) = (5u(t-1) + 10u(t) - 25u(t+2) + 15u(t+2))$ V
- → $v(t) = (5u(t+1) + 10u(t) - 25u(t-1) + 15u(t-2))$ V
- $v(t) = (5u(t-2) + 10u(t-1) - 25u(t) + 15u(t+1))$ V

Hints

Hint #1

References

Multiple Choice Difficulty: Medium Learning Objective: Understand singularity equations and their importance in solving linear differential equations.

7. Award: **10.00 points**

The voltage across a 10-mH inductor is $42\delta(t-2)$ mV. Find the inductor current, assuming that the inductor is initially uncharged.

The inductor current is $i(t)$ = 4.2 ± 2% $u(t-2)$ A.

Explanation:

The inductor current can be determined as follows:

$$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau + i(0)$$

$$i(t) = \frac{10^{-3}}{10\times10^{-3}}\int_0^t 42\delta(\tau-2)\,d\tau + 0 = 4.2u(t-2)\text{ A}$$

It should be noted that the integration of the impulse function $\delta(t-t_o)$ produces the unit step $u(t-t_o)$. Whatever the multiplier, ($f(t)$) of the impulse function at $t = t_o$ ends up multiplying the unit step by the same amount ($f(t_o)$). In this case $f(2) = 4.2$.

$$i(t) = 4.2\,u(t-2)\text{ A}$$

The inductor current is 4.2 $u(t-2)$ A.

Hints

Hint #1

References

Worksheet Difficulty: Medium Learning Objective: Understand singularity equations and their importance in solving linear differential equations.

8.

Award: **10.00 points**

Find the solution of the differential equation $\frac{dv}{dt} + 6v = 0, \;\; v(0) = -1 \text{ V}$.

The solution of the given differential equation is – (e^{-} 6 ± 2% t) V.

Explanation:

The given differential equation can be solved as follows:
$v = \text{A}\, e^{-6t}$, $v(0)$ = A = –1

Therefore, $v(t) = -e^{-6t}$ V.

The solution of the given differential equation is $-e^{-6t}$ V.

Hints

Hint #1

References

Worksheet Difficulty: Medium Learning Objective: Understand singularity equations and their importance in solving linear differential equations.

9. Award: **10.00 points**

Identify the solution of the following differential equation, subject to the stated initial condition.

$2\frac{dv}{dt} - v = 3u(t), \quad v(0) = -6$

- ○ $v(t) = -3\,(1 + e^{t/2})$ V, $t < 0$
- → ○ $v(t) = -3\,(1 + e^{t/2})u(t)$ V, $t > 0$
- ○ $v(t) = 3\,(1 - e^{t/2})u(t)$ V, $t > 0$
- ○ $v(t) = 3\,(1 - e^{t/2})$ V, $t < 0$

The given differential equation can be solved as below:

$v(t) = A + B\,e^{t/2}, \quad t > 0$

$A = -3,$

$v(0) = -6 = -3 + B \;\rightarrow\; B = -3$

$v(t) = -3\,(1 + e^{t/2})\,u(t)$ V, $t > 0$

The solution for the given differential equation is $v(t) = -3\,(1 + e^{t/2})\,u(t)$ V, $t > 0$.

Hints

Hint #1

Hint #2

References

Multiple Choice Difficulty: Medium Learning Objective: Understand singularity equations and their importance in solving linear differential equations.

A circuit is described by

$2\frac{dv}{dt} + v = 10$.

References

Section Break	Difficulty: Medium	Learning Objective: Understand singularity equations and their importance in solving linear differential equations.

10.

Award: **10.00 points**

If $v(0) = 3$, find $v(t)$ for $t \geq 0$.

The voltage $v(t)$ = 10 ± 2% + (-7 ± 2%) (e^{-} 0.50 ± 2% ^{t}) × $u(t)$ V.

Explanation:

Let $v = v_h + v_p$ and $v_p = 10$.

$$\dot{v}_h + \frac{1}{2} v_h = 0 \quad \rightarrow \quad v_h = A e^{-t/2}$$

$$v = 10 + A e^{-0.50\,t}$$

$$v(0) = 3 = 10 + A \quad \rightarrow \quad A = -7$$

$$v = 10 + (-7)\, e^{-0.50\,t}\ \text{V}$$

The voltage $v(t) = 10 + (-7)\,(e^{-0.50\,t})\,u(t)$ V.

Hints

Hint #1

Hint #2

References

Worksheet Difficulty: Medium Learning Objective: Understand singularity equations and their importance in solving linear differential equations.