A 60-μF capacitor has energy \( \omega(t) = 10 \cos^2 377t \) J and consider a positive \( v(t) \). Determine the current through the capacitor.

The current through the capacitor is \(-13.060 \pm 2\% \) \( \sin(377t) \) A.

Explanation:

\[
\omega(t) = \frac{C(v(t))^2}{2}
\]

\[
(v(t))^2 = \frac{2 \omega(t)}{C}
\]

\[
(v(t))^2 = \frac{2 \times 10 \cos^2(377t)}{60 \times 10^{-6}}
\]

\[
(v(t))^2 = 333333.3 \cos^2(377t)
\]

\[
v(t) = \pm 577.4 \cos(377t) \text{ V}
\]

Assume that \( v(t) = 577.4 \cos(377t) \) V.

\[
i(t) = C \frac{dv}{dt}
\]

\[
i(t) = 60 \times 10^{-6} \text{ F} \times 577.4 \times (-377 \sin(377t)) \text{ V}
\]

\[
i(t) = -13.060 \sin(377t) \text{ A}
\]

The current through the capacitor is \(-13.060 \sin(377t) \) A.
The voltage across a 4-$\mu$F capacitor is shown in the given figure.

Find the currents for the ranges given below.

The currents at the given ranges are as follows:
For $0 < t < 2$ ms, $i_C(t) = 20$ mA
For $2$ ms $< t < 6$ ms, $i_C(t) = -20$ mA
For $6$ ms $< t < 8$ ms, $i_C(t) = 20$ mA

Explanation:

For $0 < t < 2$ ms, $i_C(t) = 4 \times 10^{-6} \frac{d(5000t)}{dt} = 20$ mA.
For $2$ ms $< t < 6$ ms, $i_C(t) = 4 \times 10^{-6} \frac{d(20 - 5000t)}{dt} = -20$ mA.
For $6$ ms $< t < 8$ ms, $i_C(t) = 4 \times 10^{-6} \frac{d(-40 + 5000t)}{dt} = 20$ mA.

The currents at the given ranges are as follows:
For $0 < t < 2$ ms, $i_C(t) = 20$ mA
For $2$ ms $< t < 6$ ms, $i_C(t) = -20$ mA
For $6$ ms $< t < 8$ ms, $i_C(t) = 20$ mA
A capacitor has the terminal voltage

\[ v = \begin{cases} 
50 \text{ V} & t \leq 0 \\
Ae^{-100t} + Be^{-600t}v_0 & t \geq 0
\end{cases} \]

The capacitor has an initial current of 2 A.

Find the constants \( A \) and \( B \) if the capacitance is \( C = 4 \text{ mF} \).

The constants \( A \) and \( B \) are \( 61 \pm 2\% \) and \( -11 \pm 2\% \), respectively.

**Explanation:**

\[ i = C \frac{dv}{dt} = -100Ae^{-100t} - 600Be^{-600t} \]  \hspace{1cm} (1)

\[ i(0) = 2 = -100AC - 600BC \rightarrow 5 = -A - 6B \]  \hspace{1cm} (2)

\[ v(0^+) = v(0^-) \rightarrow 50 = A + B \]  \hspace{1cm} (3)

Solving (2) and (3) leads to \( A = 61 \) and \( B = -11 \).

The constants \( A \) and \( B \) are 61 and -11, respectively.
4.

A capacitor has the terminal voltage
\[ v = \begin{cases} 
50 \text{ V} & , \ t \leq 0 \\
Ae^{-100t} + Be^{-600t}V_i & , \ t \geq 0 
\end{cases} \]
The capacitor has an initial current of 2 A.

Find the capacitor current for \( t > 0 \), where the capacitance \( C = 4 \text{ mF} \).

The capacitor current is
\[ i = \boxed{-24.4 \pm 2\%}e^{-100t} + \boxed{26.4 \pm 2\%}e^{-600t} \text{ A}. \]

**Explanation:**

\[ i = (-100 \times 61 \times 4 \times 10^{-3}e^{-100t}) + (600 \times 11 \times 4 \times 10^{-3}e^{-600t}) = -24.4e^{-100t} + 26.4e^{-600t} \text{ A} \]
The capacitor current is \( i = -24.4e^{-100t} + 26.4e^{-600t} \text{ A} \).
A 4-mF capacitor has the current waveform shown in the given figure. Assume that \( v(0) = 10 \text{ V} \).

Find the value of voltage for \( 6 \text{ s} < t < 8 \text{ s} \).

The value of voltage \( v(t) = [(2.5 \pm 2\%)t - (2.5 \pm 2\%)] \text{ V} \).
Explanation:

\[ v = \frac{1}{C} \int_0^t i \, dt + v(0) = 10 + \frac{1}{4 \times 10^{-3}} \int_0^t i(t) \, dt \]

For \( 0 < t < 2 \) s, \( i(t) = 15 \) mA,
\[ v(t) = 10 + \frac{10^{-3}}{4 \times 10^{-3}} \int_0^t 15 \, dt = 10 + 3.75t \]

\[ v(2) = 10 + 7.5 = 17.5 \text{ V} \]

For \( 2 s < t < 4 s \), \( i(t) = -10 \) mA,
\[ v(t) = \frac{1}{4 \times 10^{-3}} \int_2^t i(t) \, dt + v(2) = -\frac{10 \times 10^{-3}}{4 \times 10^{-3}} \int_2^t dt + 17.5 = 22.5 - 2.5t \]

\[ v(4) = 22.5 - 2.5 \times 4 = 12.5 \text{ V} \]

For \( 4 s < t < 6 s \), \( i(t) = 0 \),
\[ v(t) = \frac{1}{4 \times 10^{-3}} \int_2^t 0 \, dt + v(4) = 12.5 \text{ V} \]

For \( 6 s < t < 8 s \), \( i(t) = 10 \) mA,
\[ v(t) = \frac{10 \times 10^{-3}}{4 \times 10^{-3}} \int_4^t dt + v(6) = 2.5(t - 6) + 12.5 = 2.5t - 2.5 \]

Hence,
\[ v(t) = [2.5t - 2.5] \text{ V} \]

The value of voltage \( v(t) = [2.5t - 2.5] \text{ V} \).
Find the voltage across the capacitors in the given circuit under dc conditions, where \( R_1 = 66 \, \Omega \) and \( R_2 = 17 \, \Omega \).

\[
\begin{array}{c}
\text{R}_1 & \quad \text{C}_1 & \quad \text{v}_1 & \quad + & \quad \text{R}_2 & \quad \text{C}_2 & \quad \text{v}_2 \\
\text{R}_1 & \quad \text{v}_1 & \quad - & \quad \text{10} \, \Omega & \quad + & \quad \text{60 V} & \quad \text{50} \, \Omega & \quad \text{v}_2 \\
\end{array}
\]

The voltage across the capacitors are \( v_1 = 42.58 \pm 2\% \) V and \( v_2 = 49.03 \pm 2\% \) V.

**Explanation:**

Under dc conditions, the circuit becomes as shown below:

\[
\begin{array}{c}
i_1 & \quad + \quad \text{10} \, \Omega & \quad i_2 & \quad \text{50} \, \Omega \\
\text{R}_1 & \quad \text{v}_1 & \quad - & \quad \text{60 V} & \quad \text{v}_2 \\
\end{array}
\]

\[
i_2 = 0 \\
i_1 = \frac{60 \, \text{V}}{66 + 10 + 17}
\]

\( i_1 = 0.65 \, \text{A} \)

\( v_1 = 66i_1 = 42.58 \, \text{V} \)

\( v_2 = 60 - 17i_1 = 49.03 \, \text{V} \)

The voltage across the capacitors are \( v_1 = 42.58 \, \text{V} \) and \( v_2 = 49.03 \, \text{V} \).
Determine the equivalent capacitance for the given circuit, where $C = 6 \, \text{F}$.

The equivalent capacitance is $4.00 \pm 2\% \, \text{F}$.

**Explanation:**

- 4 F in series with 12 F = $4 \times \frac{12}{16} = 3 \, \text{F}$
- 3 F in parallel with 6 F and 3 F = $3 + 6 + 3 = 12 \, \text{F}$
- 6 F in series with 12 F = 4.00 F
- i.e. $C_{eq} = 4.00 \, \text{F}$

The equivalent capacitance is 4.00 F.
8. value: 10.00 points

Find the equivalent capacitance in the given circuit if all capacitors are 30 μF.

The equivalent capacitance is 16.364 ± 2% μF.

Explanation:

30 μF in parallel with 30 μF = 60 μF
30 μF in series with 30 μF = 15 μF
15 μF in parallel with 30 μF = 45 μF
Hence, the circuit is reduced to that shown below.

1/C_{eq} = ((1/45 \, \mu F) + (1/45 \, \mu F) + (1/60 \, \mu F))
1/C_{eq} = 0.0611 \, \mu F
C_{eq} = 16.364 \, \mu F

The equivalent capacitance is 16.364 \, \mu F.
In the given circuit, assume that the capacitors were initially uncharged and that the current source has been connected to the circuit long enough for all the capacitors to reach steady-state (no current flowing through the capacitors). Also assume that \( R = 8 \, \text{k}\Omega \).

Determine the voltage across each capacitor.

The voltage across each capacitor is as follows:

\[
\begin{align*}
V_{10} &= 120.00 \pm 2\% \, \text{V} \\
V_{30} &= 60.00 \pm 2\% \, \text{V} \\
V_{18} &= 60.00 \pm 2\% \, \text{V} \\
V_{30} &= 24.00 \pm 2\% \, \text{V} \\
V_{20} &= 36.00 \pm 2\% \, \text{V}
\end{align*}
\]
Explanation:

Reducing the capacitance starting from right to left, In the given figure, 30 $\mu F$ is in series with 20 $\mu F$. Therefore, the equivalent capacitance is $(30 \times 20) \mu F/(30 + 20) \mu F = 12 \mu F$. In the given figure, 12 $\mu F$ is in parallel with 18 $\mu F$. Therefore, the equivalent capacitance is $(12 + 18)\mu F = 30 \mu F$.

\[ V_{10} = 15 \text{ mA} \times 8 \text{ k}\Omega = 120.00 \text{ V} \]
\[ V_{30} = 120.00/2 = 60.00 \text{ V} \]
\[ V_{18} = 120.00/2 = 60.00 \text{ V} \]
\[ V_{30} = [20 \mu F/(30+20) \mu F] \times (60.00 \text{ V}) = 24.00 \text{ V} \]
\[ V_{20} = [30 \mu F/(30+20) \mu F] \times (60.00 \text{ V}) = 36.00 \text{ V} \]

The voltage across each capacitor are as follows:

\[ V_{10} = 120.00 \text{ V} \]
\[ V_{30} = 60.00 \text{ V} \]
\[ V_{18} = 60.00 \text{ V} \]
\[ V_{30} = 24.00 \text{ V} \]
\[ V_{20} = 36.00 \text{ V} \]
In the given circuit, assume that the capacitors were initially uncharged and that the current source has been connected to the circuit long enough for all the capacitors to reach steady-state (no current flowing through the capacitors). Also assume that \( R = 8 \, \text{k}\Omega \).

Determine the energy stored in each capacitor.

The energy stored in each capacitor is as follows:

\[
\begin{align*}
w_{10} &= 72.00 \pm 2\% \, \text{mJ} \\
w_{30} &= 54.00 \pm 2\% \, \text{mJ} \\
w_{18} &= 32.40 \pm 2\% \, \text{mJ} \\
w_{30} &= 8.64 \pm 2\% \, \text{mJ} \\
w_{20} &= 12.96 \pm 2\% \, \text{mJ}
\end{align*}
\]

Explanation:

\[
\begin{align*}
w_{10} &= [0.5 \times 10 \times 120^2 \times 10^{-6} ] \, \text{J} = 72.00 \, \text{mJ} \\
w_{30} &= [0.5 \times 30 \times 60^2 \times 10^{-6} ] \, \text{J} = 54.00 \, \text{mJ} \\
w_{18} &= [0.5 \times 18 \times 60^2 \times 10^{-6} ] \, \text{J} = 32.40 \, \text{mJ} \\
w_{30} &= [0.5 \times 30 \times 24^2 \times 10^{-6} ] \, \text{J} = 8.64 \, \text{mJ} \\
w_{20} &= [0.5 \times 20 \times 36^2 \times 10^{-6} ] \, \text{J} = 12.96 \, \text{mJ}
\end{align*}
\]

The energy stored in each capacitor is as follows:

\[
\begin{align*}
w_{10} &= 72.00 \, \text{mJ} \\
w_{30} &= 54.00 \, \text{mJ} \\
w_{18} &= 32.40 \, \text{mJ} \\
w_{30} &= 8.64 \, \text{mJ} \\
w_{20} &= 12.96 \, \text{mJ}
\end{align*}
\]
The voltage across a 57-mH inductor is given by \( v(t) = [5e^{-2t} + 2t + 4] \) V for \( t > 0 \). Determine the current \( i(t) \) through the inductor. Assume that \( i(0) = 0 \) A.

The current through the inductor is
\[
i(t) = [-43.86 \pm 2\% e^{-2t} + 17.54 \pm 2\% t^2 + 70.18 \pm 2\% t + 43.86 \pm 2\%] \text{ A.}
\]

Explanation:

\[
v = L \frac{di}{dt} \rightarrow \frac{1}{L} i(t) = \frac{1}{L} \int_{0}^{t} i(\tau) d\tau + i(0)
\]

\[
i(t) = \frac{1}{57 \times 10^{-3}} \int_{0}^{t} \left(5e^{-2\tau} + 2\tau + 4\right) d\tau + 0
\]

\[
i(t) = \frac{1}{57 \times 10^{-3}} \left(\frac{5e^{-2t}}{-2} + \frac{2t^2}{2} + 4t\right)_0^t
\]

\[
i(t) = \frac{1}{57 \times 10^{-3}} \left[-2.5e^{-2t} + t^2 + 4t - (-2.5)\right]
\]

\[
i(t) = [-43.86e^{-2t} + 17.54t^2 + 70.18t + 43.86] \text{ A}
\]

The current through the inductor is \( i(t) = [-43.86e^{-2t} + 17.54t^2 + 70.18t + 43.86] \text{ A.} \)
If the voltage waveform in the given figure is applied to a 28-mH inductor, find the inductor current \( i(t) \) for \( 0 < t < 2 \) s. Assume \( i(0) = 0 \).

The inductor current for \( 0 < t < 1 \) s is \( i(t) = \left( 89.29 \pm 2\% \right) t^2 \) A.

The inductor current for \( 1 < t < 2 \) s is \( i(t) = \left[ 357.14 \pm 2\% \right] - \left[ 357.14 \pm 2\% \right] t + \left. 89.29 \pm 2\% \right] t^2 \) A.
Explaination:

\[ i(t) = \frac{1}{t} \int_0^t v(t) \, dt + i(0) \]

For \( 0 < t < 1, v = 5t \)
\[ i = \frac{1}{28 \times 10^{-3}} \int_0^t 5t \, dt + 0 \]
\[ i = \frac{5}{28 \times 10^{-3}} \left( \frac{t^2}{2} \right)_0 \]
\[ i = \frac{5}{2 \times 28 \times 10^{-3}} \cdot t^2 \]
\[ i(t) = 89.29 \cdot t^2 \text{ A} \]
\[ i(1) = 89.29 \times 1^2 = 89.29 \text{ A} \]

For \( 1 < t < 2, v = -10 + 5t \)
\[ i = \frac{1}{28 \times 10^{-3}} \int_1^t (-10 + 5t) \, dt + i(1) \]
\[ i = \frac{1}{28 \times 10^{-3}} \left( -10t + 5 \frac{t^2}{2} \right)_1 + 89.29 \]
\[ i = \frac{5}{2 \times 28 \times 10^{-3}} \left( -4t + t^2 \right)_1 + 89.29 \]
\[ i = 89.29(-4t + t^2 + 3) + 89.29 \]
\[ i = (89.29 \times -4)t + 89.29t^2 + (89.29 \times 3) + 89.29 \]
\[ i = -357.14t + 89.29t^2 + 357.14 \]
\[ i(t) = [357.14 - 357.14t + 89.29t^2] \text{ A} \]

The inductor current for \( 0 < t < 1 \) is \( i(t) = 89.29 \cdot t^2 \text{ A} \)
The inductor current for \( 1 < t < 2 \) is \( i(t) = [357.14 - 357.14t + 89.29t^2] \text{ A} \).
Consider the given circuit under dc conditions, where $R = 30 \, \Omega$.

Find the voltage $v_C$.

The voltage $v_C$ is $0 \pm 2\%$ V.

**Explanation:**

According to current division, the voltage $v_C$ is 0 V.
The voltage $v_C$ is 0 V.
Find the energy stored in the inductor.

The energy stored in the inductor is \(2.0 \pm 2\%\) J.

Explanation:

\[
W_L = \frac{1}{2} L i_L^2 = \frac{1}{2} \left(\frac{1}{2}\right)(2.8)^2 = 2.0 \text{ J}
\]

The energy stored in the inductor is 2.0 J.
Consider $L = 30 \text{ mH}$ in the given circuit and calculate the value of $R$ that will make the energy stored in the capacitor the same as that stored in the inductor under dc conditions.

The value of $R$ that will make the energy stored in the capacitor the same as that stored in the inductor under dc conditions is $13.69 \pm 2\% \ \Omega$. 
Explanation:

Under dc conditions, the circuit is equivalent to that shown below:

$$i_L = \frac{2 \Omega}{R + 2 \Omega} \times 5 \text{ A} = \frac{10}{R + 2}$$

$$v_C = Ri_L = \frac{10R}{R + 2}$$

$$w_C = \frac{1}{2} C v_C^2 = 80 \times 10^{-6} \times \frac{100R^2}{(R + 2)^2}$$

$$w_L = \frac{1}{2} L i_L^2 = \frac{1}{2} \times 30 \times 10^{-3} \text{ H} \times \frac{100}{(R + 2)^2}$$

If $w_C = w_L,$

$$80 \times 10^{-6} \times \frac{100R^2}{(R + 2)^2} = \frac{15.0 \times 10^{-8} \times 100}{(R + 2)^2}$$

$$R = \sqrt{\frac{15.0 \times 10^{-8}}{80 \times 10^{-6}}} = 13.69 \Omega$$

The value of $R$ that will make the energy stored in the capacitor the same as that stored in the inductor under dc conditions is 13.69 Ω.
Determine the equivalent inductance $L_{eq}$ at terminals $a-b$ of the given circuit, where $L = 16$ mH.

The equivalent inductance $L_{eq}$ at terminals $a-b$ of the circuit is $7.222 \pm 2\%$ mH.

**Explanation:**

\[
\frac{1}{L} = \frac{1}{60} + \frac{1}{20} + \frac{1}{30} = \frac{1}{10} \Rightarrow L = 10 \text{ mH}
\]

\[
L_{eq} = \left(10 || (16 + 10)\right) \text{ mH} = \frac{10 \times (16 + 10)}{10 + (16 + 10)} \text{ mH} = 7.222 \text{ mH}
\]

The equivalent inductance $L_{eq}$ at terminals $a-b$ of the circuit is 7.222 mH.
Find the equivalent inductance $L_{eq}$ in the given circuit, where $L = 5$ H and $L_1 = 49$ H.

The equivalent inductance $L_{eq}$ in the circuit is $4.56 \pm 2\%$ H.

**Explanation:**

The given circuit is equivalent to that shown below:

$$L_{eq} = L\|\left(\frac{L_1}{3} + \frac{2}{3}L\right) = L\|52.334 = \frac{(5\times52.334)}{(5 + 52.334)} = 4.56 \text{ H}$$

The equivalent inductance $L_{eq}$ in the circuit is 4.56 H.
Determine the equivalent inductance $L_{eq}$ that may be used to represent the inductive network of the given figure at the terminals.

The equivalent inductance $L_{eq}$ used to represent the inductive network is $6.625 \pm 2\%$ H.
Explanation:

Let \( v = L_{eq} \frac{di}{dt} \) \hspace{1cm} (1)

\( v = v_1 + v_2 = 4 \frac{di}{dt} + v_2 \) \hspace{1cm} (2)

\( i = i_1 + i_2 \rightarrow i_2 = i - i_1 \) \hspace{1cm} (3)

\( v_2 = 3 \frac{du}{dt} \quad \text{or} \quad \frac{du}{dt} = \frac{v_2}{3} \) \hspace{1cm} (4)

\( -v_2 + 2 \frac{di}{dt} + 5 \frac{dv_2}{dt} = 0 \)

\( v_2 = 2 \frac{di}{dt} + 5 \frac{dv_2}{dt} \) \hspace{1cm} (5)

Incorporating (3) and (4) into (5),

\( v_2 = 2 \frac{di}{dt} + 5 \frac{dv_2}{dt} - 5 \frac{dv_2}{dt} = 7 \frac{di}{dt} - 5 \frac{v_2}{3} \)

\( v_2 \left(1 + \frac{5}{3}\right) = 7 \frac{di}{dt} \)

\( v_2 = \frac{21}{8} \frac{di}{dt} \)

Substituting this into (2) gives

\( v = 4 \frac{di}{dt} + \frac{21}{8} \frac{di}{dt} = \frac{53}{8} \frac{di}{dt} \)

Comparing this with (1),

\( L_{eq} = \frac{53}{8} = 6.625 \text{ H} \)

The equivalent inductance \( L_{eq} \) used to represent the inductive network is 6.625 H.
The switch in the given figure has been in position A for a long time. At \( t = 0 \), the switch moves from position A to B. The switch is a make-before-break type so that there is no interruption in the inductor current. Consider the value of current \( i = 4 \text{ A} \).

Find the current \( i(t) \) for \( t > 0 \).

The current \( i(t) \) is \( 3 \pm 2\% - 7 \pm 2\% e^{-8 \pm 2\% t} \text{ A} \).

**Explanation:**

When the switch is in position A, \( i = -4 \text{ A} = i(0) \)

When the switch is in position B, \( i(\infty) = 12/4 = 3 \text{ A} \)

\[
\tau = L/R = 1/8
\]

\[
i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}
\]

\[
i(t) = (3 - 7e^{-8t}) \text{ A}
\]

The current \( i(t) \) is \( (3 - 7e^{-8t}) \text{ A} \).
The switch in the given figure has been in position A for a long time. At \( t = 0 \), the switch moves from position A to B. The switch is a make-before-break type so that there is no interruption in the inductor current. Consider the value of current \( I = 4 \) A.

Find the voltage \( v(t) \) long after the switch is in position B.

The voltage \( v(t) \) long after the switch is in position B is \( 0 \pm 2\% \) V.

Explanation:

At steady state, the inductor becomes a short circuit so that \( v = 0 \) V.

The voltage \( v(t) \) long after the switch is in position B is 0 V.