EE101 Lecture #24 May 9, 2018

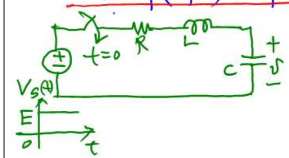
- Quiz 9 on Monday, March 12 on RLC Circuits.
- Course review on F, March 16
- Final Exam on T, March 20 4-7 p.m.

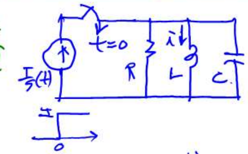
- Final Exam (12 problems)

  You are allowed to use 2 pages of formulas and tables only, not concepts, prob. soln

  Coverage: chapters 1-8, 13 (transformer, coupled inductors)
  14 (14.2 ~ 14.4 Bode plots)

Step (input) Response of RLC Circuits.





For $t > 0$

$$E = L\frac{d}{dt}\left(C\frac{dv}{dt}\right) + R\left(C\frac{dv}{dt}\right) + v$$

$$= LC\frac{d^2v}{dt^2} + RC\frac{dv}{dt} + v$$

$$\boxed{\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{1}{LC}v = \frac{E}{LC}}$$

$v(t)$ w/o input

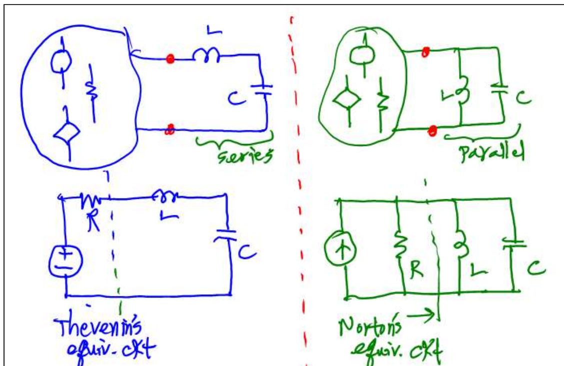
In DC steady state $v_{ss} = E$ & $v(t) = v_t + v_{ss}$

$$I_s = \frac{L\frac{di}{dt}}{R} + i + C\frac{d}{dt}\left(L\frac{di}{dt}\right)$$

$$= LC\frac{d^2i}{dt^2} + \frac{L}{R}\frac{di}{dt} + i$$

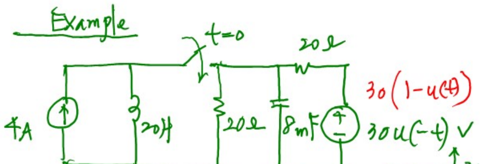
$$\boxed{\frac{d^2i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + i = \frac{I_s}{LC}}$$

Series RLC	Parallel RLC
$v(t) = v_t + v_{ss}$, $v_{ss} = E$	$i(t) = i_t(t) + i_{ss}$, $i_{ss} = I$
v_t natural response due to initial condition	$i_t(t)$ natural response due to initial condition
i) $\omega_0 > \alpha$	i) $\omega_0 > \alpha$
ii) $\omega_0 = \alpha$	ii) $\omega_0 = \alpha$
iii) $\omega_0 < \alpha$	iii) $\omega_0 < \alpha$



Series RLC | Parallel RLC

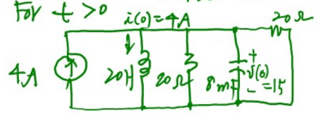
Example



$i(0_-) = 4A = i(0_+)$

$v(0_-) = 30\,\frac{20}{20+20} = 15V = v(0_+)$

For $t > 0$



$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20\times 8\times 10^{-3}}} = \frac{10}{4} = 2.5$$

$$\alpha = \frac{R}{2L} = \frac{10}{40} = 0.25$$

$\omega_0 > \alpha$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 2.4875$

$$i_t(t) = Me^{-\alpha t}\cos(\omega_d t - \theta)$$

$$i_t(t) = e^{-\alpha t}M\cos(\omega_d t - \theta) \qquad \omega_d = 2.4875$$

$$= 4.16\,e^{-0.5t}\cos(\omega_d t - 15.5^\circ)$$

$$i_t(0) = M\cos(-\theta) = M\cos\theta = 4 \qquad M = \frac{4}{0.962} = 4.16$$

$$i_t'(t)\Big|_{t=0} = -\alpha e^{-\alpha t}M\cos(\omega_d t - \theta) + e^{-\alpha t}M(-\omega_d\sin(\omega_d t - \theta))\Big|_{t=0}$$

$$= -\alpha\underbrace{M\cos\theta}_{=4} + \omega_d M\sin\theta = i_t'(0)$$

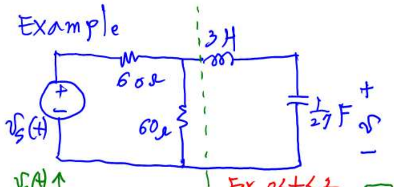
$$Li_t'(0) = v(0) = 15 \qquad i_t'(0) = \frac{15}{20} = 0.75$$

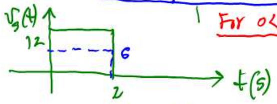
$$\frac{i_t'(0)}{4} = -\alpha + \omega\tan\theta = -0.5 + 2.4875\tan\theta$$

$$\tan\theta = \left(\frac{0.75}{4} + 0.5\right)/2.4875 = 0.276$$

$\theta = 0.27$ rad $\cos\theta = 0.962$

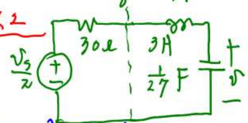
Example





For $0<t<2$

Thevenin's equiv. ckt



$\alpha = \frac{R}{2L} = \frac{30}{2\times 3} = 5$, $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3(\frac{1}{27})}} = 3$

$\alpha > 3$, thus overdamped +

$v(t) = 6 + A_1 e^{-\alpha_1 t} + A_2 e^{-\alpha_2 t}$ $s_1, s_2 = -5 \pm \sqrt{5^2-3^2} = -1, -9$

$= 6 + A_1 e^{-t} + A_2 e^{-9t}$

$v(0) = 6 + A_1 + A_2 = 0$, $v'(0) = -A_1 - 9A_2 = 0$

$v(t) = 6 + A_1 e^{-\alpha_1 t} + A_2 e^{-\alpha_2 t}$

$\frac{dv(t)}{dt} = 0 + A_1(-\alpha_1)e^{-\alpha_1 t} + A_2(-\alpha_2)e^{-\alpha_2 t} \Big|_{t=0}$

$= 0 - A_1(1)(1) + A_2(-9)(1)$

$= -A_1 - 9A_2 = v'(0)$

$C v'(0) = i(0) = 0$
capacitor current

$A_1 e^{\overset{-\alpha_1}{s_1} t} + A_2 e^{\overset{-\alpha_2}{s_2} t}$

$s^2 + 2\alpha + \omega_0^2 - \alpha^2 = 0$ $(s+\alpha)^2 - (\alpha^2 - \omega_0^2) = 0$

$(s+\alpha+\sqrt{\alpha^2-\omega_0^2}) \times (s+\alpha-\sqrt{\alpha^2-\omega_0^2}) = 0$

$-\alpha_1 = s_1 = -\alpha - \sqrt{\alpha^2-\omega_0^2} < 0$

$-\alpha_2 = s_2 = -\alpha + \sqrt{\alpha^2-\omega_0^2} < 0$

From $-A_1 - 9A_2 = 0$ $A_1 = -9A_2$

From $0 = 6 + A_1 + A_2$
$= 6 - 9A_2 + A_2$ $A_2 = \frac{3}{4}$
$A_1 = -9\left(\frac{3}{4}\right) = -\frac{27}{4}$

$v(t) = 6 - \frac{27}{4}e^{-t} + \frac{3}{4}e^{-9t}$, $v(0) = 6$

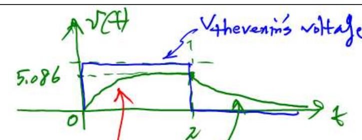
$v(2) = 6 - \frac{27}{4}e^{-2} + \frac{3}{4}e^{-18} = 5.086$, $\frac{dv}{dt} = \frac{27}{4}e^{-t} - \frac{27}{4}e^{-9t}$

$t > 2$, same circuit but
$v(t)|_{t=2} = 5.086$
no indep. voltage source

$\frac{dv}{dt}\Big|_{t=2} = \frac{27}{4}e^{-2} - \frac{27}{4}e^{-18} \approx \frac{27}{4}e^{-2}$

$$\boxed{\begin{array}{l} v(t) = \left(A_1 e^{-(t-2)} + A_2 e^{-9(t-2)}\right) u(t-2) \\ v'(t-2)\Big|_{t=2} = -A_1 - 9A_2 = \frac{27}{4}e^{-2} \end{array}}$$

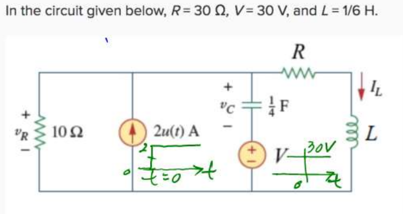
$5.086 = A_1 + A_2$
solve for A_1, A_2



$v(t) = \left[5.835 e^{-(t-2)} - 0.749 e^{-9(t-2)}\right] u(t-2)$

$v(t) = \left[-6.75 e^{-t} + 0.75 e^{-9t}\right] u(t)$

In the circuit given below, $R = 30\ \Omega$, $V = 30$ V, and $L = 1/6$ H.



Calculate $i_L(0^+)$, $v_C(0^+)$, and $v_R(0^+)$.

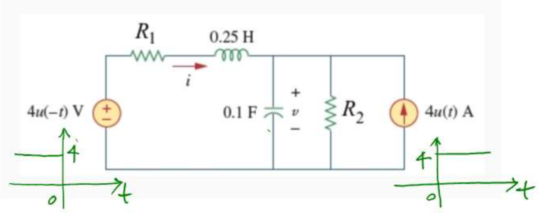
At $t = 0_-$,
inductor shorted
capacitor opened

The value of $i_L(0^+)$ is ______ A.

The value of $v_C(0^+)$ is ______ V.

The value of $v_R(0^+)$ is ______ V.

In the circuit given below, $R_1 = 4\ \Omega$ and $R_2 = 7\ \Omega$.



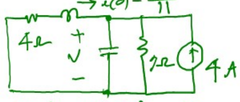
Find $dv(0^+)/dt$ and $di(0^+)/dt$.

The value of $dv(0^+)/dt$ is ______ V/s.

The value of $di(0^+)/dt$ is ______ A/s.

At $t=0$

$\rightarrow i(0) = \frac{4}{11}$



$v(t=0) = 4\left(\frac{7}{4+7}\right) = \frac{28}{11}$ ✓

$C\frac{dv(t)}{dt}\Big|_{t=0_+} = \frac{4}{11} + 4 - \frac{28}{11}/7$

$= \frac{4+16-4}{11} = \frac{16}{11}$

$\frac{dv(0_+)}{dt} = \frac{16}{11}\Big/\frac{1}{4} = \frac{64}{11}$

$4i(0) + L\frac{di(0_+)}{dt} = v(0_+)$ $\quad \frac{di(0_+)}{dt} = \frac{v(0_+) - 4i(0_+)}{L}$

$= \frac{\frac{28}{11} - 4\left(\frac{4}{11}\right)}{0.25} = \frac{12\times 4}{11}$

The current in an *RLC* circuit is described by

$\frac{d^2i}{dt^2} + 10\frac{di}{dt} + 25\,i = 0$ $\Rightarrow s^2 + 10s + 25 = (s+5)^2 = 0$

$s_{1,2} = -5$

If $i(0) = 10$ A and $di(0)/dt = 0$, then for $t > 0$, $i(t) = (\mathbf{A} + \mathbf{B}t)e^{\mathbf{s}t}$ A,

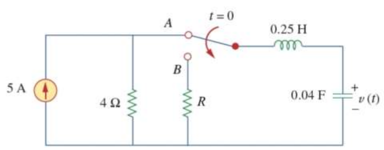
where **A** = ____, **B** = ____, and **s** = ____.

$i(0) = 10 = (A + Bt)e^{st}\Big|_{t=0} = A$

$i'(0) = Ase^{st} + Be^{st} + Btse^{st}\Big|_{t=0} = 0$

$As + B = 0 \qquad 10(-5) + B = 0 \Rightarrow B = 50$

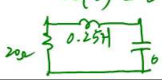
In the circuit given below, $R = 20\ \Omega$ and the switch moves (a make-before-break switch) from position *A* to *B* at $t = 0$. Find $v(t)$ for all $t \geq 0$.



The voltage equation is $v(t) = \mathbf{A}e^{\mathbf{s1}t} + \mathbf{B}e^{\mathbf{s2}t}$,

where **A** = ____, **B** = ____, **s1** = ____, and **s2** = ____.

$i(0) = 0 \qquad v(0) = 5A(4\Omega) = 20$ ✓



$\alpha = \frac{R}{2L} = \frac{20}{2(0.25)} = 40$

$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1/4\,(0.04)}} = 10$

$\alpha > \omega_0$ overdamped.

$v(t) = Ae^{-\alpha_1 t} + Be^{-\alpha_2 t}$

$\alpha_1, \alpha_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

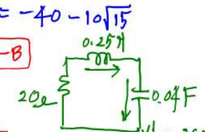
$= -40 \pm \sqrt{(40)^2 - (10)^2} = -40 \pm \sqrt{1500}$

$= -40 \pm 10\sqrt{15}$

$\alpha_1 = -40 + 10\sqrt{15}, \quad \alpha_2 = -40 - 10\sqrt{15}$

$v(0) = A + B = 20$ $\quad \boxed{A = 20 - B}$

$v'(0) = -\alpha_1 A - \alpha_2 B = 0$



$\alpha_1 A + \alpha_1 B = \alpha_1 20$

sum $(\alpha_1 - \alpha_2)B = (\alpha_1 - 1)20$

$\cancel{20}\sqrt{15}\ B = (-4) + 10\sqrt{15})\cancel{20}$

$\boxed{B = \frac{-4 + 10\sqrt{15}}{\sqrt{15}}}$

Assuming $R = 12\ \text{k}\Omega$, design a parallel *RLC* circuit that has the characteristic equation $s^2 + 100s + 10^6 = 0$.

The value of *L* is ____ H.

The value of *C* is ____ nF.

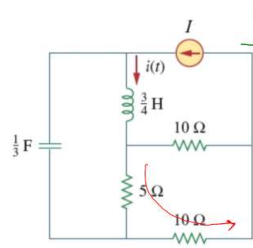
$s^2 + 100s + 10^6 = (s + 50)^2 + 10^6 - (50)^2$

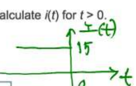
$\qquad\qquad\qquad\quad \alpha \qquad \omega_0^2 \qquad \alpha^2$

$\omega_0^2 = \frac{1}{LC} = 10^6$ $\qquad C = \frac{1}{L\,10^6} = \frac{1}{1.2K \times 10^6} = \frac{1}{1.2}\times 10^{-9}$

$\frac{R}{2L} = 5 = \frac{12K}{2L} \Rightarrow L = \frac{12K}{5\times 2} = 1.2K\ H$

In the circuit given below, $I = 15[1 - u(t)]$ A. Calculate $i(t)$ for $t > 0$.





$i(0) = 15\ A$

$i'(0) =$

$i_{5\Omega}(0) = 15 \times \frac{10}{10+15}$

$= 6\ A$

$V_{5\Omega}(0) = 5\times 6 = 30V$

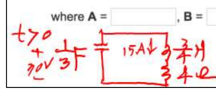
The current equation is $i(t) = [\mathbf{A}e^{\mathbf{s1}t} + \mathbf{B}e^{\mathbf{s2}t}]$ A,

$A + B = i(0) = 15$

$s_1 A + s_2 B = i'(0) = -40$

where **A** = ____, **B** = ____, **s1** = ____, and **s2** = ____.

$t > 0$



$\frac{3}{4}i'(0) + 4(15) = 30$

$i'(0) = (30 - 60)\times\frac{4}{3} = -40$