EE 101 Lecture #23 May 7, 2018

- Problem solving section on March 17. See the time and place to be announced.
- Course review on March 16 (Fri).
- Final Exam (12 problems) 3 hrs = 180 min - start

Coverage: chapters 1, 2, 13 (transistor, coupled inductors)
14 (14.2 - 14.4 state plots)

Example:

\[
\begin{align*}
R &= 40 \Omega \\
L &= 1 H \\
C &= \frac{1}{4} F
\end{align*}
\]

1. Calculate the characteristic roots of the circuit: \(\lambda^2 + 4\lambda + \frac{1}{4} = 0\)
2. Solve the characteristic equation:
\[
\lambda = -2 \pm \frac{1}{2} \sqrt{3} i
\]

\[
\begin{align*}
\lambda_1 &= -2 + \frac{1}{2} \sqrt{3} i \\
\lambda_2 &= -2 - \frac{1}{2} \sqrt{3} i
\end{align*}
\]

3. Since \(\lambda_1 \neq \lambda_2\), only one exponential term will be needed.

\[
\lambda(t) = A_1 e^{-2t} \cos \left( \frac{\sqrt{3}}{2} t \right) + A_2 e^{-2t} \sin \left( \frac{\sqrt{3}}{2} t \right)
\]

Parallel RLC Circuit

In series RLC circuit, the current is common to all elements (same).
In parallel RLC circuit, the voltage is common (same).

\[
V_{RLC} = \frac{V}{R} + \frac{1}{C} \int i dt + \frac{1}{L} \int v dt = 0
\]

\[
V_{RLC} = \frac{1}{R} \int i dt + \frac{1}{C} \int v dt + \frac{1}{L} \int i dt = 0
\]

Thus, \(i + \frac{1}{R} + \frac{1}{C} \int i dt = 0\)

\[
\frac{\dot{i} + i}{L} + \frac{1}{C} \frac{di}{dt} + \frac{1}{R} i = 0
\]

\[
\frac{\dot{i} + i}{L} + \frac{1}{C} \frac{di}{dt} + \frac{1}{R} i = 0
\]

\[
\frac{\dot{i} + i}{L} + \frac{1}{C} \frac{di}{dt} + \frac{1}{R} i = 0
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\[
\frac{\dot{i} + i}{L} + \frac{1}{C} \frac{di}{dt} + \frac{1}{R} i = 0
\]
### Dual pairs:

<table>
<thead>
<tr>
<th>Dual pair</th>
<th>Conductance $g$</th>
<th>Inductance $L$</th>
<th>Capacitance $C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage $v$</td>
<td>Current $i$</td>
<td>Voltage source</td>
<td>Current source</td>
</tr>
<tr>
<td>Node</td>
<td>Mesh</td>
<td>Series-path</td>
<td>Parallel path</td>
</tr>
<tr>
<td>Open circuit</td>
<td>Short circuit</td>
<td>KVL</td>
<td>KCL</td>
</tr>
<tr>
<td>Thevenin</td>
<td>Norton</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Two circuits that are described by equations of the same form, but in which the variables are interchanged, are said to be dual to each other.

**Two circuits are said to be duals of one another if they are described by the same characterizing equations with dual quantities interchanged.**

---

**Example:**

Given the characteristic eq. of the parallel RLC circuit as

\[
\frac{d^2}{dt^2} + \frac{1}{RC} \frac{d}{dt} + \frac{1}{L} = 0
\]

Let \( \omega_c = \frac{1}{\sqrt{LC}} \)

Case 1: \( \omega > \omega_c \), \( \frac{1}{L} > \frac{1}{RC} \)

\[
V(t) = A e^{\omega_1 t} + B e^{\omega_2 t}
\]

Case 2: \( \omega = \omega_c \), \( \frac{1}{L} = \frac{1}{RC} \)

\[
V(t) = A e^{\omega_1 t} + A \omega_1 t + B e^{\omega_2 t}
\]

---

**Parallel RLC Circuit (recalled):**

In series RLC circuit, the current is $C$ common to all elements.

In parallel RLC circuit, the node voltage is common.

\[
\begin{align*}
KCL & \Rightarrow \frac{1}{R} + \frac{1}{L} = C \frac{d}{dt} + C \frac{\Delta V}{\Delta t} = 0 \\
\Delta V & \Rightarrow \frac{\Delta V}{\Delta t} = \frac{\Delta V}{\Delta t} + \frac{L}{C} \frac{\Delta i}{\Delta t} = 0 \\
\text{Thus} & \quad \frac{d^2}{dt^2} + \frac{1}{RC} \frac{d}{dt} + \frac{1}{L} = 0
\end{align*}
\]

---

**Case 1:** \( \omega > \omega_c \), \( \Delta V = 0 \)

\[
\frac{d}{dt} \left( e^{-\omega_c t} \right) = \frac{\Delta V}{\Delta t} = 0
\]

\[
V(t) = e^{-\omega_c t} \left( A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} \right)
\]

**Case 2:** \( \omega = \omega_c \), \( \Delta V = 0 \)

\[
\frac{d^2}{dt^2} + \frac{1}{RC} \frac{d}{dt} + \frac{1}{L} = 0
\]

\[
\frac{d}{dt} \left( e^{-\omega_c t} \right) = \frac{\Delta V}{\Delta t} = 0
\]

\[
V(t) = e^{-\omega_c t} \left( A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} \right)
\]
Example

\[ V(t) = \frac{1}{L} \frac{dI}{dt} = \frac{1}{LC} \frac{dQ}{dt} = \frac{1}{LC} \sin \omega t \]

\[ I = \frac{1}{LC} \frac{dQ}{dt} = \frac{1}{LC} \sin \omega t \]

\[ \Psi(t) = C \cos \omega t \]

\[ \frac{d\Psi(t)}{dt} = -C \omega \sin \omega t \]

\[ \frac{d^2\Psi(t)}{dt^2} = -C \omega^2 \cos \omega t \]

\[ C \frac{d^2\Psi(t)}{dt^2} + M \omega^2 \Psi(t) = 0 \]

Case 2:

\[ \frac{d\Psi(t)}{dt} = C \omega \sin \omega t \]

\[ \frac{d^2\Psi(t)}{dt^2} = C \omega^2 \cos \omega t \]

\[ C \frac{d^2\Psi(t)}{dt^2} + M \omega^2 \Psi(t) = 0 \]

Step (input) Response of RLC Circuits.

\[ V(t) = \frac{1}{L} \frac{dQ}{dt} = \frac{1}{LC} \sin \omega t \]

Series RLC

\[ V(t) = V_i + V_q \]

Parallel RLC

\[ V(t) = \frac{1}{L} \frac{dQ}{dt} = \frac{1}{LC} \sin \omega t \]

Pendulum

\[ \frac{d^2\theta}{dt^2} = \frac{mgL \sin \theta}{\ell} \]

:\[ \frac{d^2\theta}{dt^2} = \frac{mgL \sin \theta}{\ell} \]

Example

\[ V(t) = \frac{1}{L} \frac{dQ}{dt} = \frac{1}{LC} \sin \omega t \]

\[ \frac{d\Psi(t)}{dt} = C \omega \sin \omega t \]

\[ \frac{d^2\Psi(t)}{dt^2} = C \omega^2 \cos \omega t \]

\[ C \frac{d^2\Psi(t)}{dt^2} + M \omega^2 \Psi(t) = 0 \]

Case 2:

\[ \frac{d\Psi(t)}{dt} = C \omega \sin \omega t \]

\[ \frac{d^2\Psi(t)}{dt^2} = C \omega^2 \cos \omega t \]

\[ C \frac{d^2\Psi(t)}{dt^2} + M \omega^2 \Psi(t) = 0 \]
\[ \ddot{x}(t) = -\frac{4}{4} M \cos(\omega t + \theta) \quad \omega = 2.4785 \]
\[ M \cos(\omega t + \theta) = 4.16 e^{0.9 t} \cos(0.5 - 15.5 \theta) \]
\[ \dot{x}_d(t) = M \cos(-\theta) = M \cos \theta = 4 \left[ \cos = \frac{4}{4} \right] \]
\[ \dot{x}_d(t) = \frac{-4}{4} M \cos(\omega t + \theta) \]
\[ \int_{0}^{t} e^{-4 t} M \cos(\omega t + \theta) dt = -0.5 M \cos \theta + 0.5 M \sin \theta = \dot{x}_d(t) \]
\[ L_d(t) = 0.25 = 0.25 \]
\[ \dot{x}(0) = 5 \quad \dot{x}(t) = 0 \]
\[ \dot{x}(0) = -5 + \omega \tan \theta = -0.97 + 2.4895 \tan \theta \]
\[ \tan \theta = (\frac{0.97}{0.25} + 0.5) \quad 2.4895 \approx 0.291 \]