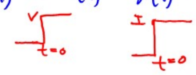
EE101 Lecture #21 May 2, 2018

- Quiz 8 on March 5, 2018

  coverage Chap. 6 & 7 First-order circuits
  - Thevenin's equivalent circuit
  - Norton's "
  - C_{eq}, L_{eq}, R_{eq}
  - solution of RC circuits for step inputs $v(t) = V\,u(t)$ or $i(t) = I\,u(t)$



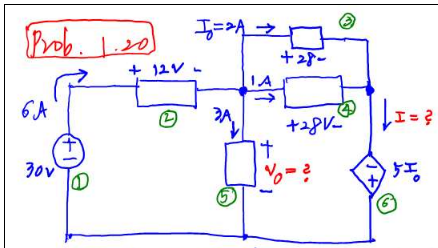
Course Grading

1) Average of best 7 Quiz scores scaled to 100 — 20%

2) Mid-term Examination — 30%

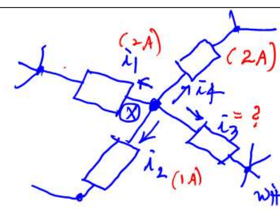
3) Final Examination — 50%
   (March 20, 2 pages of formulas allowed)

Prob. 1.20



Find the power absorbed in each element (① - ⑥)

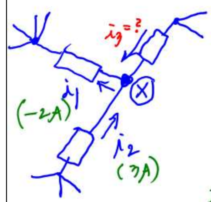
	V	I	Power absorbed
①			
②			
③			
④			
⑤			
⑥			



KCL at ⊗:

$i_1 + i_2 + i_3 + i_4 = 0$

with outgoing +



KCL at ⊗:

outgoing +	−
incoming −	+

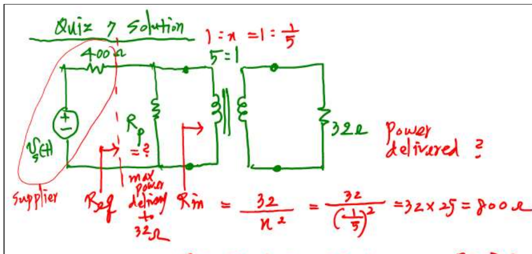
$i_1 - i_2 - i_3 = 0$

$-i_1 + i_2 + i_3 = 0$ — same

$\times(-1) \to i_1 - i_2 - i_3 = 0$

Quiz 7 solution

$1 : n = 1 : \frac{1}{5}$



$R_{in} = \frac{32}{n^2} = \frac{32}{(\frac{1}{5})^2} = 32 \times 25 = 800\,\Omega$

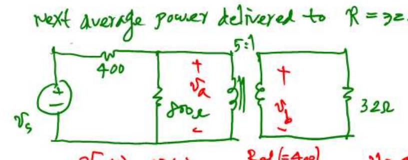
$R_{eq} = R_p \parallel R_{in} = R_p \parallel 800\,\Omega = \frac{R_p\, 800}{R_p + 800}$

For max. power transfer, $R_s = R_{eq}$ or

$400 = \frac{R_p \cdot 800}{R_p + 800} \Rightarrow (R_p + 800)\cancel{400} = R_p\, \cancel{800}$

thus $\boxed{R_p = 800\,\Omega}$

next average power delivered to $R = 32\,\Omega$?



$v_a(t) = v_s(t) \frac{R_{eq}(=400)}{400 + (R_{eq} = 400)} = \frac{110 \sin \omega t}{2} = 55 \sin \omega t$

$v_b(t) = \frac{1}{5} v_a(t) = 11 \sin \omega t$

$P_{32\Omega}(t) = \frac{v_b^2(t)}{32\,\Omega} = \frac{11^2 \sin^2 \omega t}{32} = \frac{121}{32} \sin^2 \omega t$

$\sin^2 \omega t = \frac{1}{2} - \frac{1}{2} \cos 2\omega t$

$P_{32}^{avg} = \frac{1}{T} \int_0^T \frac{121}{32} \sin^2 \omega t\, dt = \frac{121}{32} \frac{1}{T} \int_0^T \left(\frac{1}{2} - \cancel{\frac{1}{2} \cos 2\omega t}\right) dt$

$= \frac{121}{32 \times 2} = \boxed{1.89\ \text{W}}$

" Science News "

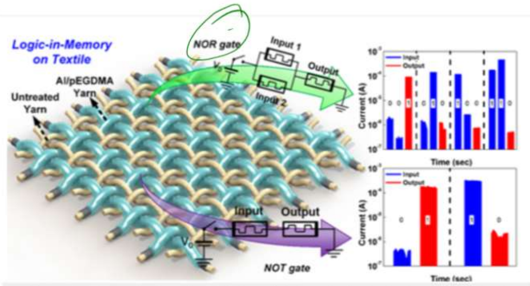


Figure 3. Schematic view of the fabricated device with logic circuits and electrical measured data of the NOR and NOT gate on fabric.

PEGDMA = poly Ethylene Glycol Dimeta Acrylate

Functional Circuitry on Commercial Fabric via Textile-Compatible Nanoscale Film Coating Process for Fibertronics

Hagyoul Bae†, Byung Chul Jang†§, Hongkeun Park‡, Soo-Ho Jung‖, Hye Moon Lee‖, Jun-Young Park†, Seung-Bae Jeon†, Gyeongho Son†, Il-Woong Tcho†, Kyoungsik Yu†, Sung Gap Im‡, Sung-Yool Choi*†§, and Yang-Kyu Choi*†

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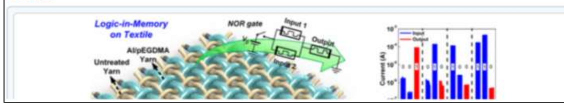
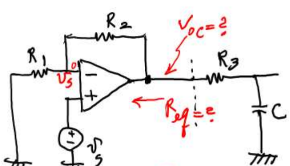
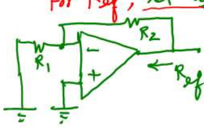
Abstract





$$V_{oc} = \left(\frac{v_s}{R_1}\right) R_2 + v_s = v_s\left(1 + \frac{R_2}{R_1}\right)$$

For R_{eq}, set v_s (indep. source) = 0

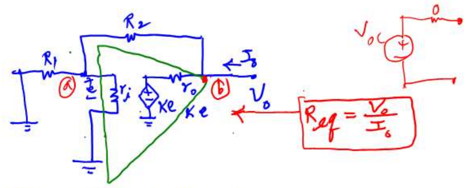


R_2 ? ~~X~~

$R_1 + R_2$? $\cancel{X}$

Something else ? ___

What about the path through Op Amp to ground ?



$$R_{eq} = \frac{V_o}{I_o}$$

KCL at (b) : $\frac{e - V_o}{R_2} + \frac{Ke - V_o}{r_o} + I_o = 0$ (1)

express e in terms of V_o

$$e = V_o\left(\frac{R_1 \| r_i}{R_2 + R_1 \| r_i}\right) = \alpha V_o \quad (2)$$

From (1) & (2),

$$I_o = \frac{V_o - \alpha V_o}{R_2} + \frac{V_o - K\alpha V_o}{r_o} = \left[\frac{1-\alpha}{R_2} + \frac{1 - K\alpha}{r_o}\right] V_o$$

$$\Rightarrow \frac{V_o}{I_o} = \frac{1}{\frac{1-\alpha}{R_2} + \frac{1-K\alpha}{r_o}} = \frac{R_2 r_o}{(1-\alpha) r_o + (1 - K\alpha) R_2}$$

For $e = 0$ $\Rightarrow \alpha = 0$

$R_{eq} = \frac{R_2 r_o}{r_o + R_2} = R_2 \| r_o$

Example $R_1 = R_2 = 10K$

$r_i = 10\ M\Omega$, $r_o = 100\Omega$, $K = 10^6$

$$R_{eq} = \frac{R_2 r_o}{(1-\alpha) r_o + (1 - K\alpha) R_2} = \frac{10K\,(100)}{(1-0.5)100 + (1 - 10^6 0.5) 10K}$$

$$\alpha = \frac{R_1 \| r_i}{R_2 + R_1 \| r_i} = \frac{\frac{(10K)(10M)}{10K + 10M}}{10K + \frac{(10K)(10M)}{10K + 10M}} = 0.5$$

$= \frac{R_1}{R_1 + R_2}$ for $r_i \gg R_1$

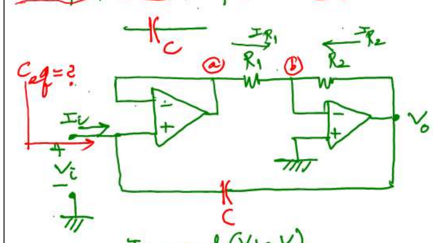
$$= \frac{10K\,(100)}{50 + (1 - 0.5 \times 10^6) 10K}$$

$$\cong \frac{100}{-0.5 \times 10^6} = -200\ \mu\Omega \approx 0$$

$$R_{eq} = \frac{R_2 r_o}{(1-\alpha) r_o + (1 - K\alpha) R_2} = \frac{r_o}{(1-\alpha)\frac{r_o}{R_2} + (1 - K\alpha)} = \boxed{-\frac{r_o}{\alpha K}}$$

$K\alpha \gg 1$
$R_2 \gg r_o$
$(1-\alpha)\frac{r_o}{R_2} \approx 0$

Capacitance Multiplier



$$I_i = C\,\frac{d(V_i - V_o)}{dt}$$

the voltage at node (a) is V_i $\Rightarrow I_{R_1} = \frac{V_i}{R_1}$

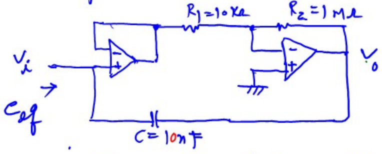
the voltage at node (b) is 0 (virtual ground)

$I_{R_2} = \frac{V_o}{R_2}$

KCL at (b) $\frac{V_i}{R_1} + \frac{V_o}{R_2} = 0 \Rightarrow V_o = -\frac{R_2}{R_1} V_i$

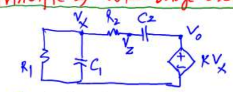
$$\Rightarrow I_i = C \frac{d}{dt}\left(V_i - \left(-\frac{R_2}{R_1} V_i\right)\right) = \boxed{C\left(1 + \frac{R_2}{R_1}\right)} \frac{dV_i}{dt}$$

Example



$$C_{eq} = C\left(1+\frac{R_2}{R_1}\right) = 10\,nF\left(1+\frac{1000K}{10K}\right)$$
$$= 1010\,nF = 1.01\,\mu F.$$

Principle of Wien-Bridge Oscillator

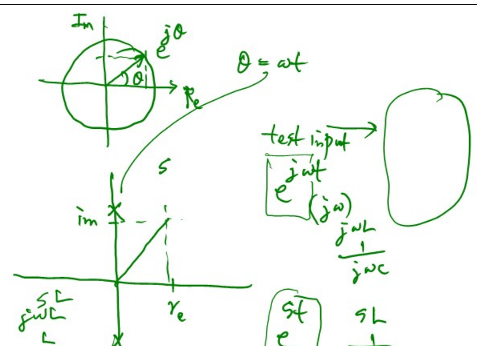


elements time domain	Impedance s-domain	Freq domain
R	R	R
L	Ls	$j\omega L$
C	$\frac{1}{Cs}$	$\frac{1}{j\omega C}$

In s-domain

$$V_X(s)\left(\frac{1}{R_1}+C_1 s\right) = \frac{(K-1)V_X(s)}{R_2+\frac{1}{C_2 s}}$$
$$V_X(s)\left(\frac{1}{R_1}+C_1 s\right)\left(R_2+\frac{1}{C_2 s}\right) = (K-1)V_X(s)$$
$$\times R_1 \cdot C_2 s \Rightarrow V_X(s)(1+R_1C_1 s)(R_2C_2 s+1) = (K-1)V_X(s)R_1C_2 s$$
$$(R_1C_1+R_2C_2)s + R_1R_2C_1C_2 s^2 + 1 = (K-1)R_1C_2 s$$
$$R_1R_2C_1C_2 s^2 + \big(\underbrace{R_1C_1+R_2C_2-(K-1)R_1C_2}_{\text{set to zero}}\big)s + 1 = 0$$

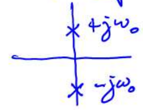
Barkhausen criterion



$$L\frac{di}{dt} + Ri$$
$$= L\frac{d\,e^{j\omega t}}{dt} + Re^{j\omega t} = j\omega L\, e^{j\omega t} + R\, e^{j\omega t} = V$$

then $R_1R_2C_1C_2 s^2 + 1 = 0$

$$s = \pm\sqrt{\frac{-1}{R_1R_2C_1C_2}} = \pm j\sqrt{\frac{1}{R_1R_2C_1C_2}} = \pm j\omega_o$$

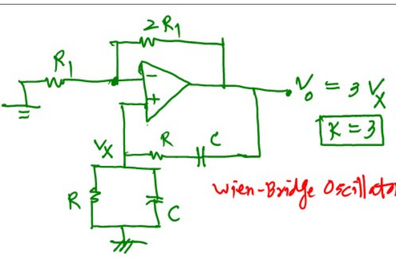


$$R_1C_1 + R_2C_2 - (K-1)R_1C_2 = 0$$
$$R_1C_1 + R_2C_2 = (K-1)R_1C_2$$
$$\boxed{K = \frac{R_1C_1+R_2C_2}{R_1C_2} + 1}$$

Barkhausen Criterion is met with this K value!

If $R_1 = R_2 = R$, $C_1 = C_2 = C$, then

$$K = \frac{2RC}{RC} + 1 = 3$$

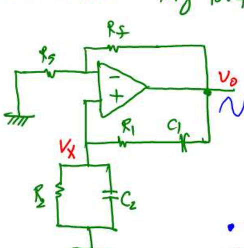


$R = 1\,K\Omega$, $C = 1\,nF$

$$\text{then } \omega_o = \frac{1}{\sqrt{R_1R_2C_1C_2}} = \frac{1}{\sqrt{R.R.C.C}} = \frac{1}{RC} = \frac{1}{10^3\times 10^{-9}}$$
$$= 10^6$$

$$\boxed{\begin{array}{ll} V_X(t) = A\cos 10^6 t & V_X(0) = A \\ V_o = 3V_X = 3A\cos 10^6 t & \end{array}}$$

In the textbook Fig 10.42



- Set $R_f = 2R_s$
- $R_1 = R_2 = R$
- $C_1 = C_2 = C$

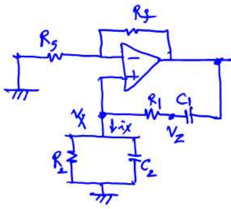
then $V_o = 3V_X$ $(K=3)$

- $\omega_o = \frac{1}{RC}$

For $R = 10\,K\Omega$ $C = 159\,\mu F$
$R_s = 10\,K\Omega$ $(R_F = 20\,K\Omega = 2\times R_s)$

$$f_o = \frac{1}{2\pi RC} = \frac{1}{2\pi(10\times 10^3)(159\times 10^{-6})}$$
$$= 100\,\times Hz$$

$V_o = V_{max}\cos(\omega_o t + \phi)$, $\omega_o = 2\pi f_o$



$V_o = K V_x$

$K = V_x\left(1 + \frac{R_g}{R_x}\right)$

$$i_x = \frac{V_x}{R_2} + C_2 \frac{dV_x}{dt} = C_1 \frac{d}{dt}\left(K V_x - V_z\right) \qquad (1)$$

$$V_z = V_x + \left(\frac{V_x}{R_2} + C_2 \frac{dV_x}{dt}\right) R_1 \qquad (2)$$

From (1) & (2)

$$\frac{V_x}{R_2} + C_2 \frac{dV_x}{dt} = C_1 \frac{d}{dt}\left[(K-1)V_x - \left(\frac{V_x}{R_2} + C_2 \frac{dV_x}{dt}\right) R_1\right]$$

$$\times R_2 \rightarrow V_x + R_2 C_2 \frac{dV_x}{dt} = C_1 (K-1) R_2 - C_1 R_1 \frac{dV_x}{dt} - R_1 R_2 C_1 C_2 \frac{d^2 V_x}{dt^2}$$

which is $R_1 R_2 C_1 C_2 \frac{d^2 V_x}{dt^2} + \left(R_1 C_1 + R_2 C_2 - C_1 (K-1) R_2\right) \frac{dX}{dt} + V_x = 0$

If we set the first derivative part to zero,

i.e. $\boxed{R_1 C_1 + R_2 C_2 - (K-1) R_2 C_1 = 0}$ (Barkhausen criterion),

then $R_1 R_2 C_1 C_2 \frac{d^2 V_x}{dt^2} + V_x = 0 \qquad (*)$

$V_x = A\cos(\omega_0 t + \phi)$ can satisfy the equation (*)

$$-R_1 R_2 C_1 C_2 \omega_0^2 A \cos(\omega_0 t + \phi) + A\cos(\omega_0 t + \phi) = 0$$

$$\Rightarrow R_1 R_2 C_1 C_2 \omega_0^2 = 1$$

$$\omega_0^2 = \frac{1}{R_1 R_2 C_1 C_2}$$

$\boxed{\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}}$, $\boxed{\omega_0 = \frac{1}{RC}}$ if $R_1 = R_2 = R$, $C_1 = C_2 = C$

$V_x = A\cos(\omega_0 t + \phi)$, $V_o = K V_x = \left(1 + \frac{R_g}{R_s}\right) \times A\cos(\omega_0 t + \phi)$