EE101 Lecture 19 Feb 26, 2018

- Quiz 6 Average 8.0, $\sigma = 1.52$
- QUIZ 7 Today
- HW 8 (additional first order ckt problems)

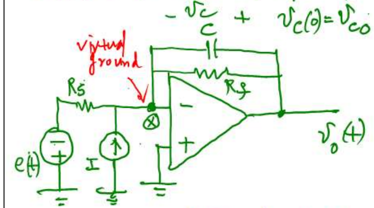
First, A method for "analog computing"
solving an ordinary differential equation

$$\frac{dx}{dt} + a_1 x + a_2 = f(t)$$

$$x(0) = x_0$$

solve for $x(t)$ for given x_0, $f(t)$

Consider the following Op Amp circuit.



$x_0 = v_{C0}$

$x(t) = v_o(t)$

KCL at node ⊗

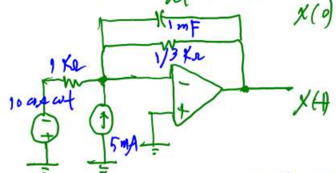
$$C\frac{dv_o}{dt} + \frac{v_o}{R_f} + I = \frac{e(t)}{R_s}$$

- $\frac{dv_o}{dt} + \frac{v_o}{R_f C} + \frac{I}{C} = \frac{e(t)}{C R_s}$
- $\frac{dx}{dt} + a_1 x + a_2 = f(t)$

$a_1 = \frac{1}{R_f C}$, $a_2 = \frac{I}{C}$, $f(t) = \frac{e(t)}{C R_s}$

Example $\frac{dx}{dt} + 3x + 5 = 10\cos\omega t$

$x(0) = 3$ v



$a_1 = \frac{1}{R_f C}$

$a_2 = \frac{I}{C}$

$f(t) = \frac{e(t)}{C R_s}$

Let $R_f = \frac{1}{3}$ kΩ, $C = 1$ mF, then $a_1 = 3$

$I = a_2 C = 5\times 10^{-3} = 5$ mA

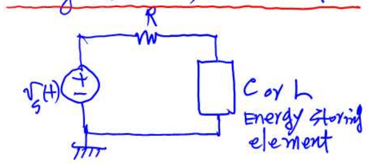
$e(t) = f(t)\, C R_s$

$= 10\cos\omega t\,(10^{-3})(10^{3})$

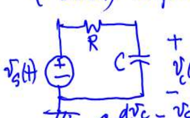
$R_s = 1$k

$= 10\cos\omega t.$

A generic RC, RL circuit



(case 1) Capacitor

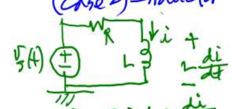


$$C\frac{dv_c}{dt} = \frac{v_s - v_c}{R}$$

$$C\frac{dv_c}{dt} + \frac{v_c}{R} = \frac{v_s}{R}$$

- $\frac{dv_c}{dt} + \left(\frac{1}{RC}\right)v_c = \frac{v_s}{RC}$

(Case 2) Inductor

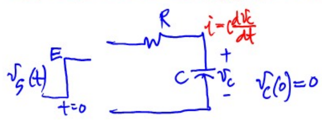


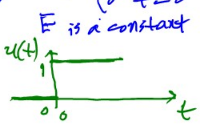
$$v_s = Ri + L\frac{di}{dt}$$

- $\frac{di}{dt} + \left(\frac{R}{L}\right)i = \frac{v_s}{L}$

A case with $v_s(t) = E\,u(t)$, $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t \le 0 \end{cases}$

E is a constant





For $t > 0$, $RC\frac{dv_c}{dt} + v_c = E$

$\Rightarrow RC\frac{dv_c}{dt} + (v_c - E) = 0$

$\Rightarrow \frac{dv_c}{dt} + \frac{1}{RC}(v_c - E) = 0$

$\frac{dv_c}{v_c - E} + \frac{1}{RC}dt = 0 \Rightarrow \frac{d(v_c - E)}{v_c - E} = -\frac{1}{RC}dt$

Integrating both sides

$$\ln(v_c - E)\Big|_{v_{c0}-E}^{v_c - E} = -\frac{t}{RC}$$

$$\boxed{\frac{v_c - E}{v_{c0} - E} = e^{-\frac{t}{RC}}} = \begin{cases} 1 & t = 0 \\ 0 & t \to \infty \end{cases}$$

$\Rightarrow \ln\frac{v_c - E}{v_{c0} - E} = -\frac{t}{RC}$

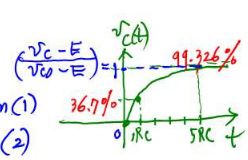
$v_c(t) - E = (v_{c0} - E)\,e^{-\frac{t}{RC}}$

- $v_c(t) = E + (v_{c0} - E)\,e^{-\frac{t}{RC}}$ (1)

$v_c(\infty) = E$

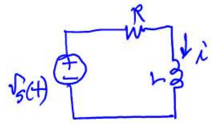
$RC = \tau$ (time constant)

$e^{-\frac{t}{\tau}}$	$\frac{t}{\tau}$
0.36788	1
	⋮
0.00674	5



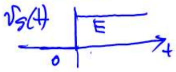
- If $E = 0$, $v_{c0} \neq 0$, then from (1)

$\boxed{v_c(t) = v_{c0}\,e^{-t/RC}}$ (2)



$v_S(t) = Ri + L\frac{di}{dt}$

For $v_S(t)$



For $t > 0$

$L\frac{di}{dt} + Ri = E$

$L\frac{di}{dt} + Ri - E = 0$

$\left(\frac{L}{Ri-E}\right)di + dt = 0$

$\frac{L}{R}\frac{d(Ri-E)}{(Ri-E)} + dt = 0$

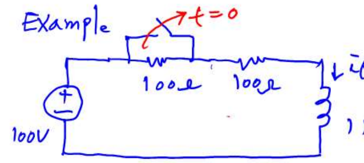
Integrating over $t=0,t$ $\Rightarrow \ln(Ri-E)\Big|_{Ri_0-E}^{Ri-E} + \frac{R}{L}t = 0$

$\ln\frac{Ri-E}{Ri_0-E} + \frac{R}{L}t = 0$

$\Rightarrow Ri(t) - E = (Ri_0 - E)e^{-\frac{R}{L}t}$

$i(t) = \frac{E}{R} + \left(i_0 - \frac{E}{R}\right)e^{-t/(\frac{L}{R})}$ $\boxed{\tau = \frac{L}{R}}$ (3)

Example



For $t \le 0$, $i(t) = 1A$, $i(0) = 1A$

For $t > 0$,

$200i + 1\frac{di}{dt} = 100$

$i(t) = \frac{E}{R} + \left(i_0 - \frac{E}{R}\right)e^{-t/(L/R)}$

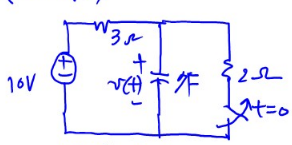
$= \frac{100}{200} + \left(1 - \frac{100}{200}\right)e^{-t/(1/200)}$

$= 0.5 + 0.5e^{-200t}$

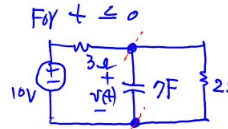
$i(0) = 0.5 + 0.5 = 1$ ✓

$i(\infty) = 0.5$ ✓

(Example)



For $t \le 0$



$v_{OC} = 10\frac{2}{3+2} = 4V$

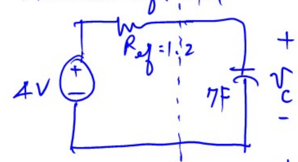
$R_{eq} = 3 \| 2 \quad \frac{3\times2}{3+2} = 1.2\Omega$

From (2) for v_{C0}, $e^{-t/RC}$

$v(t) = v_{C0}e^{-t/RC}$

$= 4e^{-t/(1.2\times7)}$

Thevenin's eq. ckt



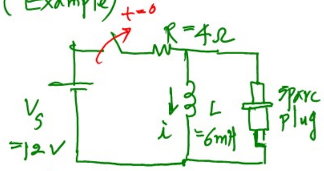
$v_{C0} = 4V$

$v_C(t) = E + (v_{C0} - E)e^{-t/RC}$

$= 4 + \underbrace{(4-4)}_{=0}e^{-t/RC}$

$= 4$? double check

(Example)



$t_{switch\ to\ open} = 1\ \mu sec$

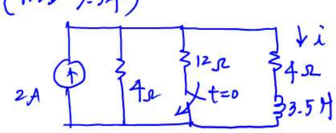
For $t \le 0$, $i = \frac{12V}{4\Omega} = 3A$

the voltage across the spark is

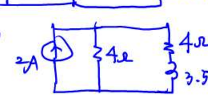
$L\frac{di}{dt} = L\frac{\Delta i}{\Delta t} = 6\times10^{-3}\frac{(3-0)A}{1\times10^{-6}s}$

$= 18\ KV$

(Prob 7.54) 5th edition



For $t \le 0$



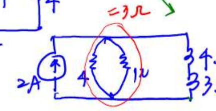
From (2)

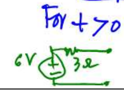
$i(t) = \frac{E}{R} + \left(i_0 - \frac{E}{R}\right)e^{-\frac{t}{L/R}}$

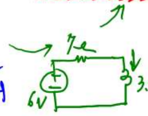
$= \frac{6}{7} + \left(1 - \frac{6}{7}\right)e^{-\frac{t}{3.5/7}}$

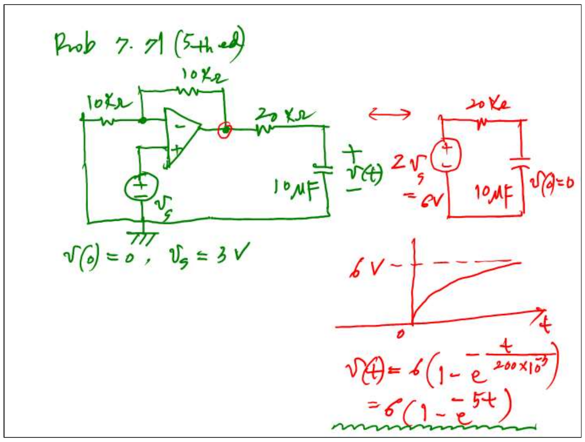
$= \frac{6}{7} + \frac{1}{7}e^{-\frac{t}{2}}$

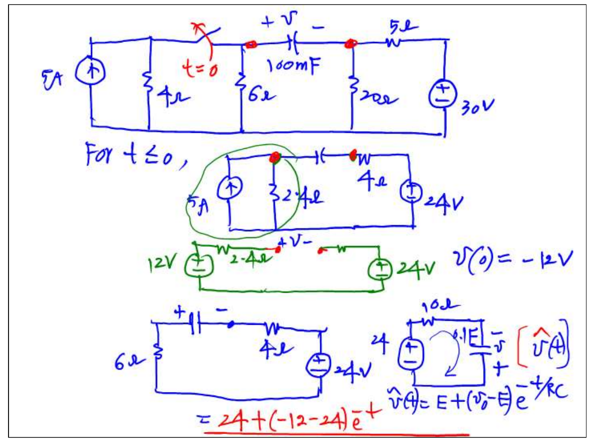
For $t > 0$








Prob 7.71 (5th ed)
10kΩ
10kΩ
20kΩ
10μF
v(t)
v(0) = 0, v_s = 3V
2v_s = 6V
20kΩ
10μF
v(0) = 0
6V
v(t) = 6(1 − e^(−t/(200×10^−3)))
= 6(1 − e^(−5t))


t=0
+ v −
100mF
5Ω
5A
4Ω
6Ω
20Ω
30V
For t ≤ 0,
5A
2.4Ω
4Ω
24V
12V
2.4Ω
+ v −
24V
v(0) = −12V
6Ω
4Ω
24V
10Ω
0.1F
24
v̂(t)
v̂(t) = E + (v₀ − E)e^(−t/RC)
= 24 + (−12 − 24)e^(−t)