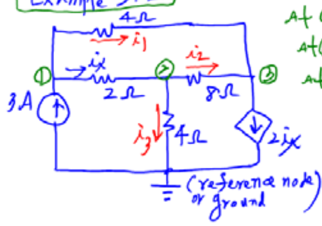
# EE101 Lecture #7 Jan 24, 2018

Further Discussion on Circuit Analysis
(Nodal Analysis – KCL
Loop/Mesh Analysis – KVL)

Example 3.2




At ① $3 = i_1 + i_x$
At ② $i_x = i_2 + i_3$
At ③ $i_1 + i_2 = 2i_x$
} (A)

$i_1 = \frac{v_1 - v_3}{4}$
$i_2 = \frac{v_2 - v_3}{8}$
$i_3 = \frac{v_2}{4}$
$i_x = \frac{v_1 - v_2}{2}$
} (B)

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From (A) & (B)

$\frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2} = 3 \Rightarrow \boxed{3v_1 - 2v_2 - v_3 = 12}$ (1)

$\frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2}{4} \Rightarrow \boxed{4v_1 - 7v_2 + v_3 = 0}$ (2)

$\frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = v_1 - v_2 \Rightarrow \boxed{-2v_1 + 3v_2 - v_3 = 0}$ (3)

In Matrix Form,

$$\underset{\sim}{A}\begin{bmatrix} 3 & -2 & -1 \\ 4 & -7 & +1 \\ -2 & +3 & -1 \end{bmatrix} \underset{\sim}{v}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \underset{\sim}{b}\begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$v_k = \frac{\Delta_k}{\Delta}$

$\Delta = |A|$

$\Delta_k = |A_k|$

where $A_k$ = A with kth column replaced with $\underset{\sim}{b}$

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$$v_2 = \frac{\begin{vmatrix} 3 & 12 & -1 \\ 4 & 0 & +1 \\ -2 & 0 & -1 \end{vmatrix}}{\begin{vmatrix} 3 & -2 & -1 \\ 4 & -7 & +1 \\ -2 & 3 & -1 \end{vmatrix}} = \frac{0 + 0 - 24 - 0 - 0 - (-48) = 24}{21 + (-12) + 4 - (-14) - 9 - 8 = 10}$$

$= 2.4\,[V]$

1 2 3 + + +

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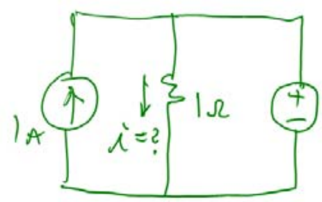
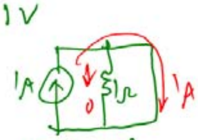
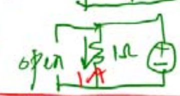
# Superposition




Case 1 Let 1V → 0




Case 2 Let 1A → 0




Solution 0 + 1 = 1 A

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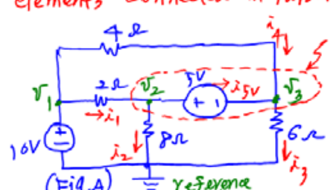
# Nodal Analysis with a Supernode

A supernode is formed by enclosing an independent or a dependent voltage source between 2 non-reference nodes and any elements connected in parallel with it.




(Fig A)

$i_1 - i_2 - i_{5V} = 0$
$i_4 + i_{5V} - i_3 = 0$
sum $i_1 - i_2 + i_4 - i_3 = 0$

same ⟸


(Fig B)

$i_1 + i_4 - i_2 - i_3 = 0$

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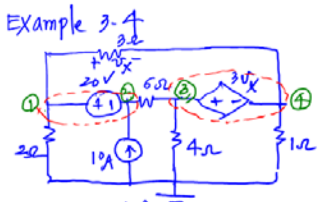
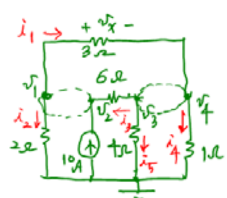
Example 3.4




(4 nodes)




4 KCL eqs
4 unknowns
$(v_1, v_2, v_3, v_4)$

• $SN_1$: $i_3 + 10 = i_1 + i_2$

$\frac{v_3 - v_2}{6} + 10 = \frac{v_1 - v_4}{3} + \frac{v_1}{2}$

$\boxed{5v_1 + 2v_2 - v_3 - 2v_4^{*} = 60}$ (1)

$v_1 = 20 + v_2$ (2) and $v_4 = v_3 - 3v_x = v_3 - 3(v_1 - v_4)^{*}$ (3)

• $SN_2$: $i_1 = i_3 + i_4 + i_5 \rightarrow \frac{v_1 - v_4}{3} = \frac{v_3 - v_2}{6} + \frac{v_3}{4} + \frac{v_4}{1}$

$\Rightarrow \boxed{4v_1 + 2v_2 - 5v_3 - 16v_4^{*} = 0}$ (4)

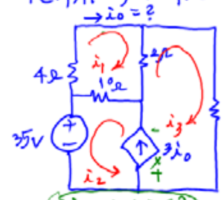
In summary

$$\begin{bmatrix} 5 & 2 & -1 & -2 \\ 4 & 2 & -5 & -16 \\ 1 & -1 & 0 & 0 \\ -3 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ 20 \\ 0 \end{bmatrix}$$

$$\underset{\sim}{A} \qquad \underset{\sim}{v} \qquad \underset{\sim}{b}$$

$$v_k = \frac{|A_k|}{|\underset{\sim}{A}|} \quad \text{where } \underset{\sim}{A_k} = A \Big| \text{kth column replace by } \underset{\sim}{b}$$

Revisit of a Previous Prob. solved




Our solution was $i_0 = i_1 = \frac{105}{104}$ [A]

we said the voltage across CCCS was x and removed it by subtracting one eq. from the other.

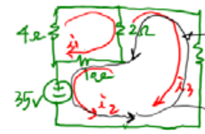
loop 1 $\quad 4i_1 + 2(i_1 - i_3) + 10(i_1 + i_2) = 0 \quad (1)$

loop 2 $\quad 35 + 10(i_1 + i_2) + x = 0 \quad (2)$

loop 3 $\quad 2(i_3 - i_1) + 8i_3 + x = 0 \quad (3)$

$3i_1 = i_2 + i_3 \ (4)$

$(2) - (3) \Rightarrow \quad 35 + 10(i_1 + i_2) - 2(i_3 - i_1) - 8i_3 = 0 \quad (5)$




This is the loop eq. for a combined loop! obtained by removing the (dependent) current source branch

combined loop

In summary

$$\underbrace{\begin{bmatrix} 16 & 10 & -2 \\ 12 & 10 & -10 \\ -3 & 1 & 1 \end{bmatrix}}_{\underset{\sim}{A}} \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}}_{\underset{\sim}{i}} = \underbrace{\begin{bmatrix} 0 \\ -35 \\ 0 \end{bmatrix}}_{\underset{\sim}{b}}$$

$$i_1 = i_o = \frac{|A_1|}{|A|} = \frac{420}{416} = \frac{105}{104} [A]$$

Ans. same as in the previous solution (Lect 6)

$$|A| = \begin{vmatrix} 16 & 10 & -2 \\ 12 & 10 & -10 \\ -3 & 1 & 1 \end{vmatrix} = 416$$

$$|A_1| = \begin{vmatrix} 0 & 10 & -2 \\ -35 & 10 & -10 \\ 0 & 1 & 1 \end{vmatrix} = 420$$