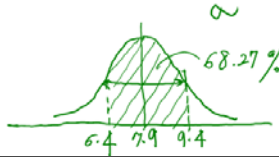
EE 101 Lecture #6 Jan 22, 2018

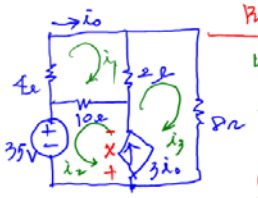
★ Midterm date change

From Feb 7 (W) to ★ Feb 12 (M) (new date)

1 Page of formulas/tables allowed.

Quiz #1 Average ≈ 7.9 / 10.0, 1.49





Problem: Find i_o

Loop 1: $4i_1 + 2(i_1 - i_3) + 10(i_1 + i_2) = 0$ (1)

Loop 2: $35 + 10(i_1 + i_2) + X = 0$ (2)

Loop 3: $2(i_3 - i_1) + 8i_3 + X = 0$ (3)

$i_o = i_1$

Let's try to remove X

(2) − (3) → $35 + 10(i_1 + i_2) + X - 2(i_3 - i_1) - 8i_3 - X = 0$

$35 + 12i_1 + 10i_2 - 10i_3 = 0$ (4)

Also from CCCG, $i_2 + i_3 = 3i_o = 3i_1$ (5)

From (5) & (4), $35 + 12i_1 + 10i_2 - 10(3i_1 - i_2) = 0$ ($i_o = i_1$)

or $35 - 18i_1 + 20i_2 = 0$ (6)

From (5) & (1) →

$4i_1 + 2(i_1 - (3i_1 - i_2)) + 10(i_1 + i_2) = 0$

or $10i_1 + 12i_2 = 0$ (7)

In summary

$$\begin{aligned} -18i_1 + 20i_2 &= -35 \quad (6) \\ 10i_1 + 12i_2 &= 0 \quad (7) \end{aligned}$$

From (7) $i_2 = -\frac{10}{12} i_1 = -\frac{5}{6} i_1$ (8)

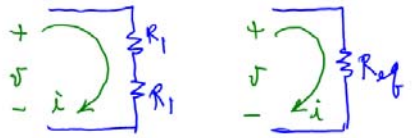
From (6) & 8

$-18i_1 + 20\left(-\frac{5}{6} i_1\right) = -35$

$\left(-18 - \frac{50}{3}\right) i_1 = -35$

$i_1 = i_o = \frac{35}{18 + \frac{50}{3}} = \frac{105}{104}$

Equivalent Resistance

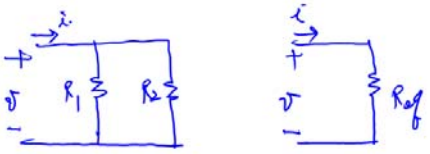


$v = R_1 i + R_2 i$

$= (R_1 + R_2) i$

$v = R_{eq} i$

$R_{eq} = R_1 + R_2$ (serial connection)



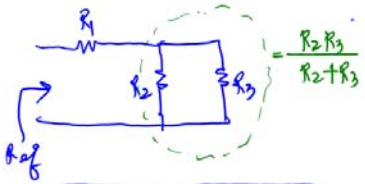
$i = \frac{v}{R_1} + \frac{v}{R_2}$

$= v\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$i = \frac{v}{R_{eq}}$

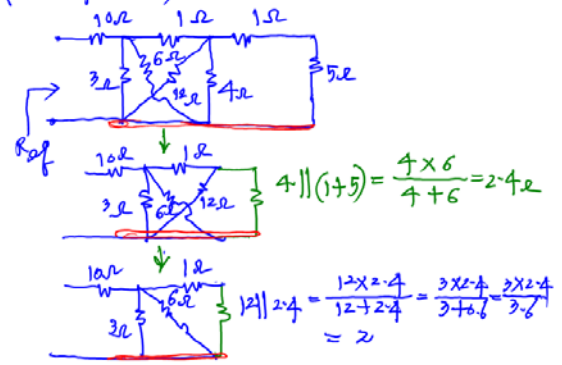
$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$

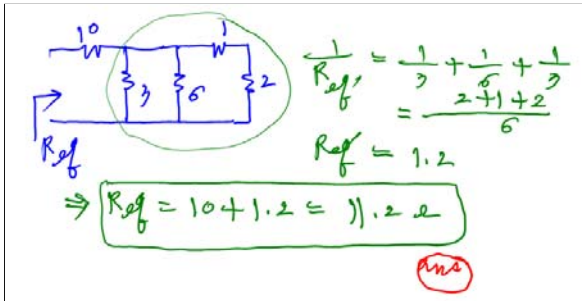
$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$ (parallel connection)



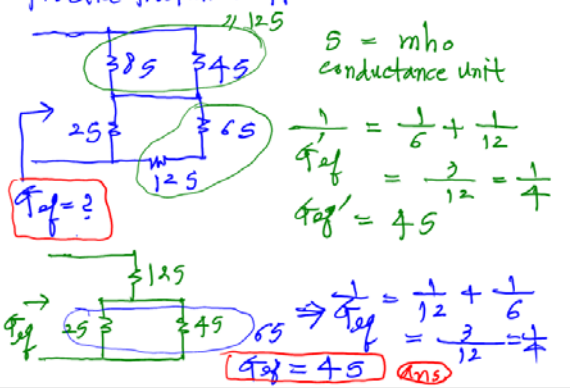
$R_{eq} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$

(Example 2.10)

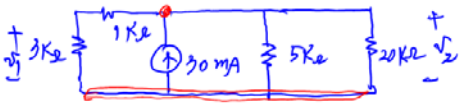




Practice Problem 2.11



(Practice Prob. 2.13



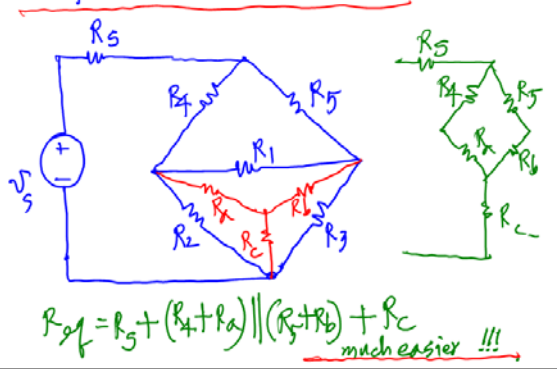
KCL at node X

$30mA = v_2\left(\frac{1}{4K\Omega} + \frac{1}{5K\Omega} + \frac{1}{20K\Omega}\right) = v_2\left(\frac{5+4+1}{20K\Omega}\right)$

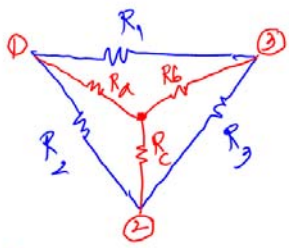
$v_2 = 30mA \times 2K\Omega = 60V$

$v_1 = \frac{60V}{4K\Omega} \times 3K\Omega = 45V$

Wye - Delta Transformation



$R_{eq} = R_5 + (R_4 + R_a) \| (R_5 + R_b) + R_c$ much easier !!!



$R_{12\,eq} = R_2 \| (R_1 + R_3) = R_a + R_c$ (1)

$R_{13\,eq} = R_1 \| (R_2 + R_3) = R_a + R_b$ (2)

$R_{23\,eq} = R_3 \| (R_1 + R_2) = R_b + R_c$ (3)

$(2)-(1) \Rightarrow R_b - R_c = R_1 \| (R_2+R_3) - R_2 \| (R_1+R_3)$ (3)

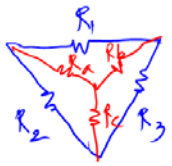
$R_b + R_c = R_3 \| (R_1+R_2)$ (4)

$(3)+(4) \Rightarrow 2R_b = R_1 \| (R_2+R_3) - R_2 \| (R_1+R_3) + R_3 \| (R_1+R_2)$

$$R_b = \frac{1}{2}\frac{R_1(R_2+R_3) - R_2(R_1+R_3) + R_3(R_1+R_2)}{R_1+R_2+R_3}$$

$$= \frac{1}{2}\frac{R_1R_2 + R_1R_3 - R_2R_1 - R_2R_3 + R_1R_3 + R_2R_3}{R_1+R_2+R_3}$$

$$= \boxed{\frac{R_{1\Delta}\cdot R_{3\Delta}}{R_{1\Delta}+R_{2\Delta}+R_{3\Delta}} = R_{bY}}$$



$$R_{bY} = \frac{R_{1\Delta}R_{3\Delta}}{R_{1\Delta}+R_{2\Delta}+R_{3\Delta}}$$

$$R_{aY} = \frac{R_{1\Delta}R_{2\Delta}}{R_{1\Delta}+R_{2\Delta}+R_{3\Delta}}$$

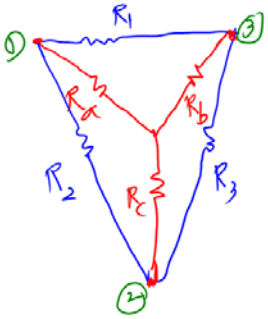
$$R_{cY} = \frac{R_{2\Delta}R_{3\Delta}}{R_{1\Delta}+R_{2\Delta}+R_{3\Delta}}$$

$$R_{aY}R_{bY} + R_{bY}R_{cY} + R_{cY}R_{aY} = \frac{R_{1\Delta}R_{2\Delta}R_{1\Delta}R_{3\Delta} + R_{1\Delta}R_{2\Delta}R_{2\Delta}R_{3\Delta} + R_{2\Delta}R_{3\Delta}R_{1\Delta}R_{2\Delta}}{(R_{1\Delta}+R_{2\Delta}+R_{3\Delta})^2} = \frac{R_{1\Delta}R_{2\Delta}R_{3\Delta}(R_{1\Delta}+R_{2\Delta}+R_{3\Delta})}{(R_{1\Delta}+R_{2\Delta}+R_{3\Delta})^2} = \frac{R_{1\Delta}R_{2\Delta}R_{3\Delta}}{R_{1\Delta}+R_{2\Delta}+R_{3\Delta}}$$

$$R_{1\Delta} = \frac{R_{aY}R_{bY} + R_{bY}R_{cY} + R_{cY}R_{aY}}{R_{cY}}$$

$$R_{2\Delta} = \frac{R_{aY}R_{bY} + R_{bY}R_{cY} + R_{cY}R_{aY}}{R_{bY}}$$

$$R_{3\Delta} = \frac{R_{aY}R_{bY} + R_{bY}R_{cY} + R_{cY}R_{aY}}{R_{aY}}$$



wye to Delta

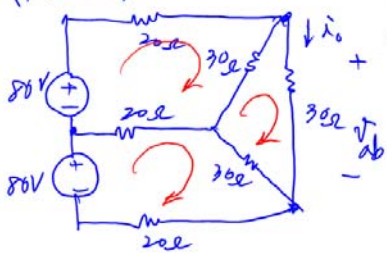
Y → Δ

$$R_{1\Delta} = \frac{R_{aY}R_{bY} + R_{bY}R_{cY} + R_{cY}R_{aY}}{R_{cY}}$$

$$R_{2\Delta} = \frac{R_{aY}R_{bY} + R_{bY}R_{cY} + R_{cY}R_{aY}}{R_{bY}}$$

$$R_{3\Delta} = \frac{R_{aY}R_{bY} + R_{bY}R_{cY} + R_{cY}R_{aY}}{R_{aY}}$$

(Prob 3.43)

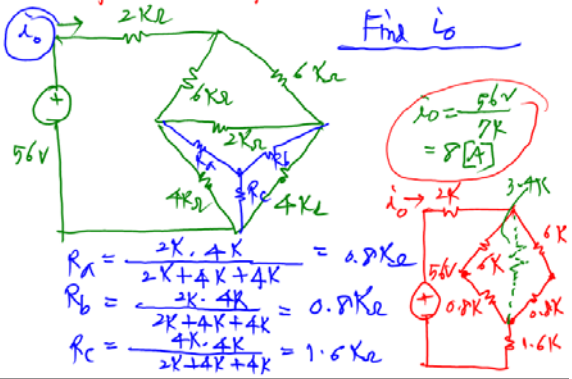


Find v_{ab} & i_o

use Mesh Analysis

example of Δ−Y transformation

Find i_o



$$R_A = \frac{2K\cdot 4K}{2K+4K+4K} = 0.8K\Omega$$

$$R_b = \frac{2K\cdot 4K}{2K+4K+4K} = 0.8K\Omega$$

$$R_c = \frac{4K\cdot 4K}{2K+4K+4K} = 1.6K\Omega$$