EE101 Lecture #3 Jan 12, 2018

sewebsite https://courses.soe.ucsc.edu/courses/ee101/winter18/01

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Text C.K. Alexander & M.N.O. Sadiku Fundamentals of Electric Circuits

<connect>
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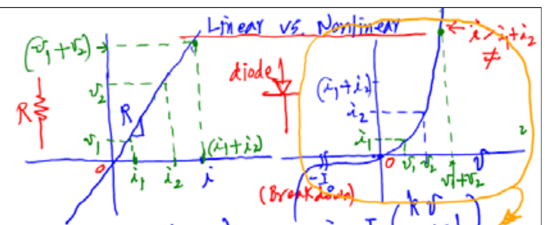
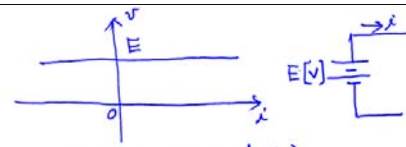


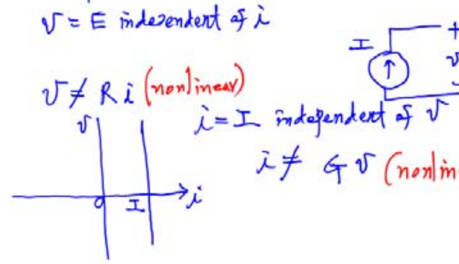
$v = R\,i$ (linear)
$v_1 = R i_1$
$v_2 = R i_2$
$(v_1+v_2) = R(i_1+i_2)$

$i = I_0(e^{kv}-1)$
nonlinear
$i_1 = I_0(e^{kv_1}-1)$
$i_2 = I_0(e^{kv_2}-1)$
$I_0(e^{k(v_1+v_2)}-1) > (i_1+i_2)$ $\neq$



$v = E$ independent of i

$v \neq R i$ (nonlinear)



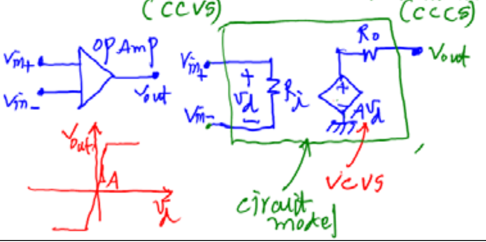
$i = I$ independent of v

$i \neq G v$ (nonlinear)

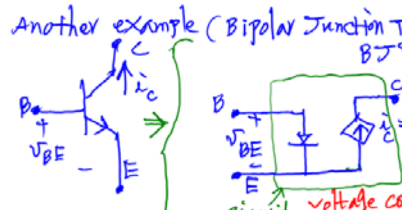
Dependent Sources

dependent voltage source
- voltage controlled (VCVS)
- current controlled (CCVS)

dependent current source
- voltage controlled (VCCS)
- current controlled (CCCS)



Another example (Bipolar Junction Transistor)

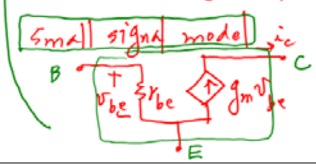


$i_c = I_s e^{\frac{q v_{BE}}{kT}}$
$= f(v_{be})$

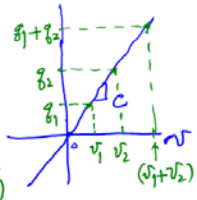
voltage controlled current source (VCCS)

NPN type BJT

Small signal model

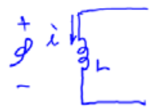


$q = Cv$



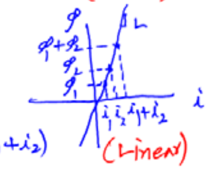
$q_1 + q_2 = Cv_1 + Cv_2$
$= C(v_1 + v_2)$

(Linear)



$\varphi = Li$

$\varphi_1 = Li_1$
$\varphi_2 = Li_2$
$\varphi_1 + \varphi_2 = L(i_1 + i_2)$



(Linear)

In general

$y = f(x)$ f linear function

$y_1 = f(x_1)$, $y_2 = f(x_2)$

For $x = ax_1 + bx_2$

$y = f(ax_1 + bx_2)$

$\overset{\uparrow}{=}\ af(x_1) + bf(x_2) = ay_1 + by_2$

(f linear)

Complex number

$Z = x + jy$

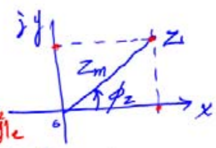
magnitude, phase angle

$Z = Z_m \angle \phi_Z$

$= \sqrt{x^2 + y^2} \angle \tan^{-1}\frac{y}{x} = \phi_Z$

$= Z_m \cos\phi_Z + jZ_m \sin\phi_Z$

$Z = Z_m e^{j\phi_Z}$

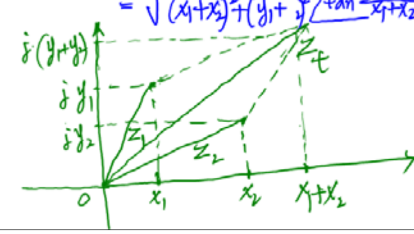


$Z_1 = x_1 + jy_1 = Z_{1m} \angle \phi_{Z_1}$

$Z_2 = x_2 + jy_2 = Z_{2m} \angle \phi_{Z_2}$

$Z_t = Z_1 + Z_2 = (x_1 + x_2) + j(y_1 + y_2)$

$= \sqrt{(x_1+x_2)^2 + (y_1+y_2)^2} \angle \tan^{-1}\frac{y_1+y_2}{x_1+x_2} = Z_{tm} \angle \phi_{Z_t}$



$v = V_m \cos(\omega t + \phi_v)$

$= \mathrm{Re}\left[V_m e^{j(\omega t + \phi_v)}\right]$

$\overset{\uparrow}{=} \mathrm{Re}\left[V_m\left(\cos(\omega t + \phi_v) + j\sin(\omega t + \phi_v)\right)\right]$

$(e^{j\theta} = \cos\theta + j\sin\theta)$

$= \mathrm{Re}\left[V_m e^{j\phi_v} e^{j\omega t}\right] = \mathrm{Re}\left[\mathbb{V} e^{j\omega t}\right]$

$\boxed{\mathbb{V} = V_m e^{j\phi_v} = V_m \angle \phi_v}$

phasor

Likewise

$I = I_m \cos(\omega t + \phi_I)$

$= \mathrm{Re}\left[I_m e^{j(\omega t + \phi_I)}\right]$

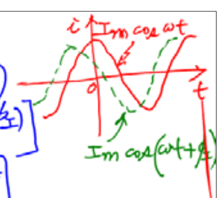
$= \mathrm{Re}\left[I_m e^{j\phi_I} e^{j\omega t}\right]$

$= \mathrm{Re}\left[\mathbb{I} e^{j\omega t}\right]$

$\boxed{\mathbb{I} = I_m e^{j\phi_I} = I_m \angle \phi_I}$

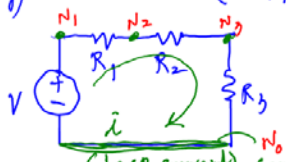
phasor

$\mathbb{V} \cdot \mathbb{I} = V_m e^{j\phi_v} \cdot I_m e^{j\phi_I} = V_m I_m e^{j(\phi_v + \phi_I)}$



Linear elements

e.g.) Linear Resistors (series connection)



Loop Analysis
Mesh analysis

In this case loop analysis is preferred: only one unknown for given V, R_1, R_2, R_3

$$V \overset{\text{ohm's Law}}{=} R_1 i + R_2 i + R_3 i = (R_1 + R_2 + R_3) i$$

$$i = V/(R_1 + R_2 + R_3)$$

e.g. Linear Resistors (Parallel Connection)

In this case Nodal Analysis is better than Loop Analysis.

$$I = i_1 + i_2 + i_3 \overset{\text{Ohm's Law}}{=} \frac{V_{N_1}}{R_1} + \frac{V_{N_1}}{R_2} + \frac{V_{N_1}}{R_3} = V_{N_1}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$V_{N_1} = I \Big/ \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

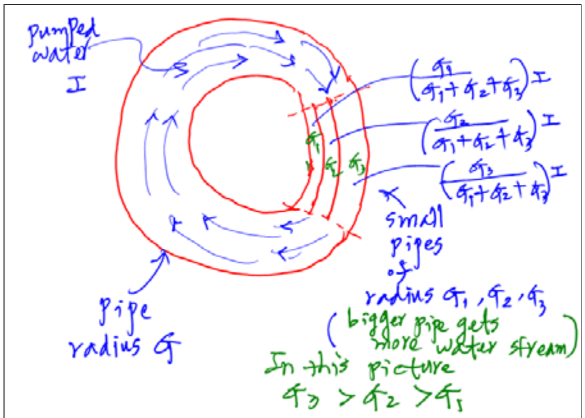
In Ohm's Law $v = R i$, R = resistance $[\Omega]$ Ohm

or $i = \frac{v}{R} = \frac{1}{R} v = G v$

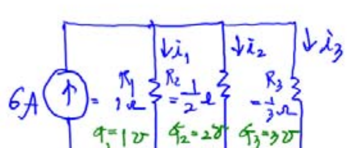
G is conductance $[\mho]$ mho

So $V_{N_1} = I \Big/ \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = I/(G_1 + G_2 + G_3)$

$$i_1 = \frac{V_{N_1}}{R_1} = G_1 V_{N_1} = I\left[\frac{G_1}{G_1 + G_2 + G_3}\right]$$



Revisit of the Example

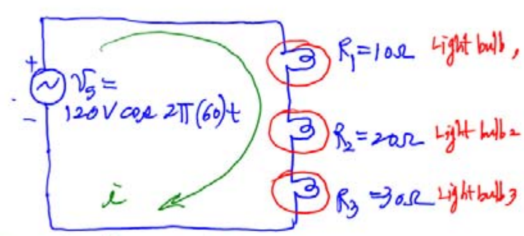


$$i_1 = I\frac{G_1}{G_1 + G_2 + G_3} = 6\,\frac{1}{1+2+3} = 1\,[A]$$

$$i_2 = I\frac{G_2}{G_1 + G_2 + G_3} = 6\,\frac{2}{1+2+3} = 2\,[A]$$

$$i_3 = I\frac{G_3}{G_1 + G_2 + G_3} = 6\,\frac{3}{1+2+3} = 3\,[A]$$

$I = i_1 + i_2 + i_3$ (✓) (KCL)



Loop (Mesh) current i = ?

By Ohm's Law $i = \frac{V_s}{R_1 + R_2 + R_3} = \frac{120 \cos 120\pi t}{60}$

$= 2 \cos 120\pi t\ [A]$

Average power in Light Bulb 1

$$P_1 = \frac{1}{T}\int_0^T v_1(t)\, i(t)\, dt$$

$$= \frac{1}{T}\int_0^T R\, i^2(t)\, dt$$

$v_1 = R_1 i$

$$= \frac{10}{T}\int_0^T \left[2\cos 120\pi t\right]^2 dt$$

$$= \frac{10\times 4}{T}\int_0^T \cos^2 120\pi t\, dt$$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

$$= \frac{40}{T}\int_0^T \frac{1}{2}\left(1+\cos 2(120\pi t)\right) dt$$

$$= \frac{40}{T}\left[\frac{1}{2}T + \underbrace{\frac{1}{2}\int_0^T \cos 2(120\pi t)\, dt}_{=0}\right]$$

$$= 20\,W$$

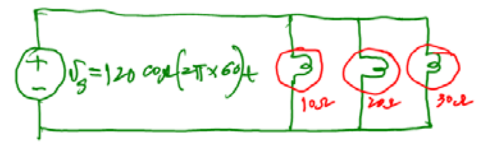
For Ligh Bulb 2 ($R_2 = 20\Omega$)

$$P_2 = \frac{20}{10}\left[20W\right] = 40\,W$$

$$P_3 = \frac{30}{10}\left[20W\right] = 60\,W$$

Total wattage $P_1 + P_2 + P_3 = \boxed{120\,W}$

If bulbs are connected in Parallel



Avg Power in bulb 1

$$P_1 = \frac{1}{T}\int_0^T v_s(t)\, i_1(t)\, dt = \frac{1}{T}\int_0^T \frac{v_s^2(t)}{R_1}\, dt$$

$$= \frac{1}{10\,T}\int_0^T \left[120\cos 120\pi t\right]^2 dt$$

$$= \frac{14400}{10T}\left[\frac{1}{2}\int_0^T (1+\cos 240\pi t)\, dt\right]$$

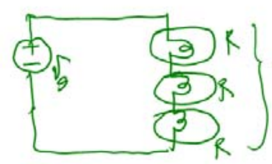
$$= \frac{144}{T}\left[\frac{1}{2}T + \underbrace{\frac{1}{2}\int_0^T \cos 240\pi t\, dt}_{=0}\right]$$

$$= 72\,W$$

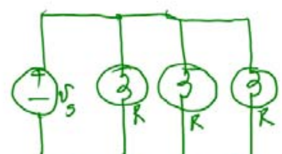
$$P_2 = 72\,W\left(\frac{10}{20}\right) = 36\,W \quad \left(P(t) = \frac{v_s^2(t)}{R}\right)$$

$$P_3 = 72\,W\left(\frac{10}{30}\right) = 24\,W$$

$$P_1 + P_2 + P_3 = 72 + 36 + 24 = \boxed{132\,W}$$



$R_T = 3R \quad P(t) = \frac{v_s^2(t)}{3R}$



$$P(t) = \frac{v_s^2(t)}{R}\times 3$$

9 time more wattage!

(Light Bulb at home are connected this way)

But if too many in parallel, then overload (shut off)