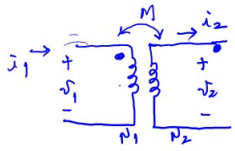
EE 101 Lecture 18, Feb. 23, 2018

Quiz 4 on Monday, Feb. 26
(HW 6 & Magnetically Coupled Circuits
★ Lecture note
[HW 7 is for RC, RL circuit analysis and would be covered in Quiz 8]

Midterm papers to be returned on Monday, Feb. 26.

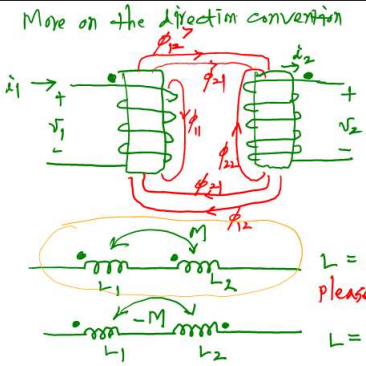


• convention
1) If both currents enters into • terminals, M is +
2) If either current enters in opposite direction, M is −
3) If both currents leaves • terminal, M is +.

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} L_1 & M_{12} \\ M_{21} & L_2 \end{bmatrix} \begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{bmatrix}$$

where $M_{12} = M_{21} = M$

More on the direction convention



Total flux

$\lambda_1 = \lambda_{11} + \lambda_{12}$

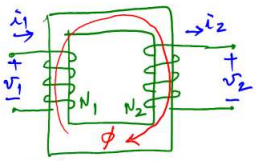
$\lambda_2 = \lambda_{21} + \lambda_{22}$

$\lambda_{ij} > 0$ when aiding, < 0, opposing

$L = L_1 + L_2 + 2M$

please note "2"

$L = L_1 + L_2 - 2M$



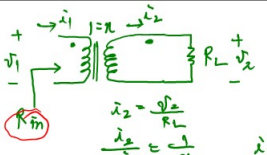
$$v_1 = N_1 \frac{d\phi}{dt}$$

$$v_2 = N_2 \frac{d\phi}{dt}$$

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n$$

$v_1 i_1 = v_2 i_2 \Rightarrow \frac{v_2}{v_1} = \frac{i_1}{i_2} = \frac{N_2}{N_1}$ (Power conservation)

$$\frac{i_2}{i_1} = \frac{N_1}{N_2} = \frac{1}{n}$$



$i_2 = \frac{v_2}{R_L}$

$\frac{i_2}{i_1} = \frac{1}{n}$

$i_1 = n i_2$

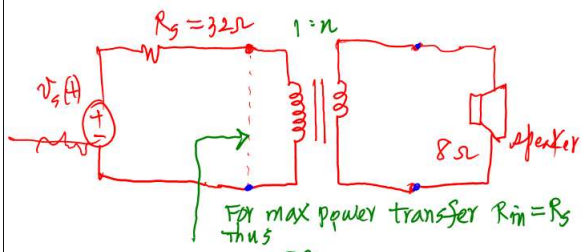
$v_1 = R_{in} i_1 = R_{in} n i_2$

$= R_{in} n \frac{v_2}{R_L} = R_{in} n \frac{n v_1}{R_L}$

$= \frac{R_{in}}{R_L} n^2 v_1$

$\Rightarrow 1 = \frac{R_{in}}{R_L} n^2$ & $\boxed{R_{in} = \frac{R_L}{n^2}}$

A case for Max. power Transfer



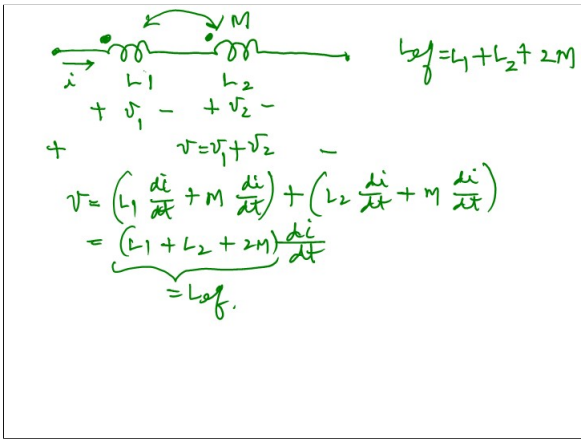
$R_s = 32\Omega$, $1:n$, 8Ω, speaker

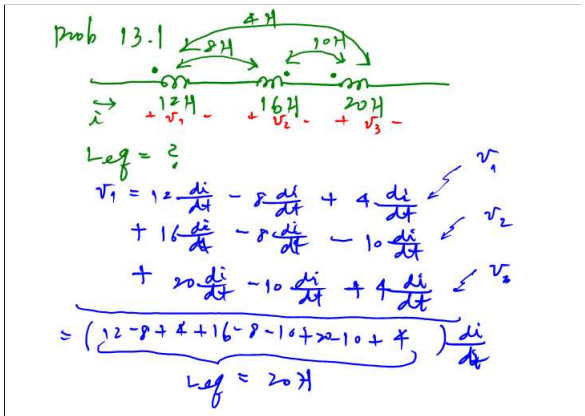
For max power transfer $R_{in} = R_s$

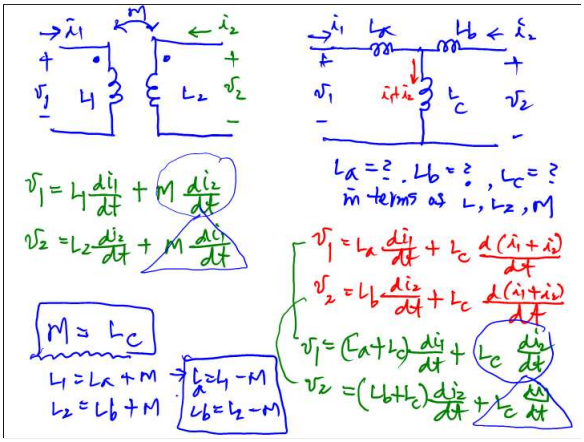
Thus

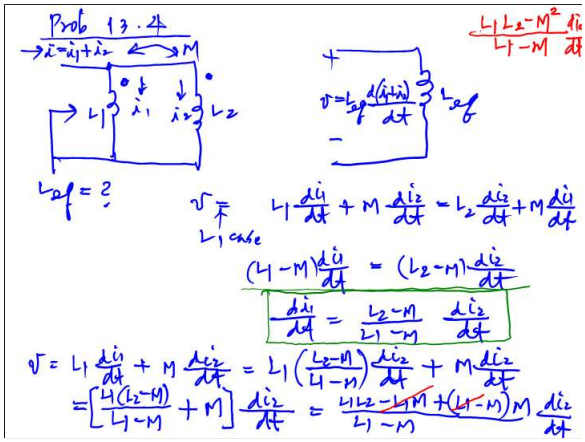
$R_{in} = \frac{8}{n^2} = 32$

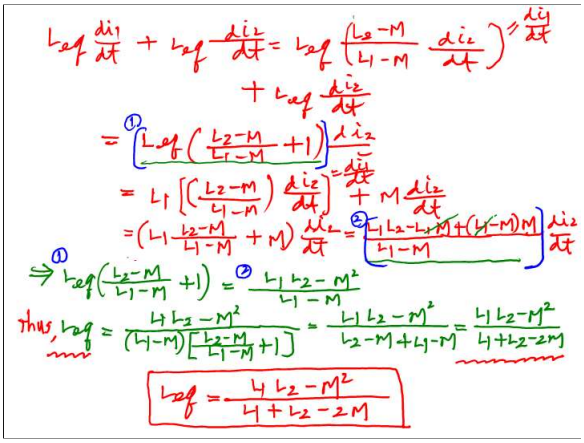
$n^2 = \frac{8}{32} = \frac{1}{4} \Rightarrow \boxed{n = \frac{1}{2}}$

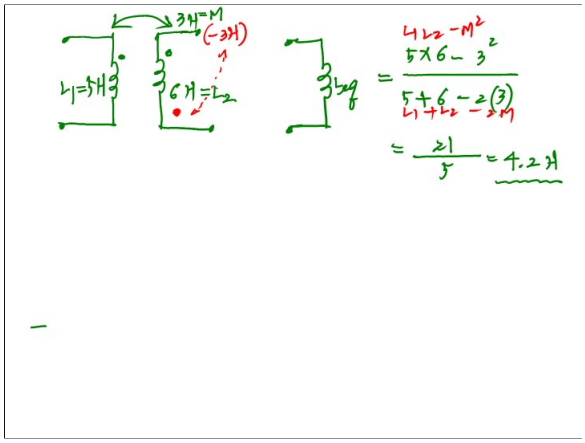

M
i
L1
L2
+ v1 -
+ v2 -
+ v -
v = v1 + v2
v = (L1 di/dt + M di/dt) + (L2 di/dt + M di/dt)
= (L1 + L2 + 2M) di/dt
= Leq.
Leq = L1 + L2 + 2M


Prob 13.1
4H
2H
10H
12H
16H
20H
i
+ v1 -
+ v2 -
+ v3 -
Leq = ?
v1 = 12 di/dt - 8 di/dt + 4 di/dt
+ 16 di/dt - 8 di/dt - 10 di/dt
+ 20 di/dt - 10 di/dt + 4 di/dt
= (12 - 8 + 4 + 16 - 8 - 10 + 20 - 10 + 4) di/dt
Leq = 20H

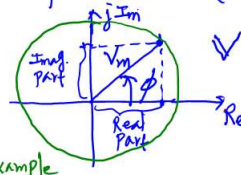

i1
i2
M
L1
L2
+ v1 -
+ v2 -
v1 = L1 di1/dt + M di2/dt
v2 = L2 di2/dt + M di1/dt
M = Lc
L1 = La + M
L2 = Lb + M
La = L1 - M
Lb = L2 - M
La = ? , Lb = ? , Lc = ?
in terms of L1, L2, M
i1
La
Lb
i2
Lc
i1 + i2
v1 = La di1/dt + Lc d(i1 + i2)/dt
v2 = Lb di2/dt + Lc d(i1 + i2)/dt
v1 = (La + Lc) di1/dt + Lc di2/dt
v2 = (Lb + Lc) di2/dt + Lc di1/dt


Prob 13.4
i = i1 + i2
M
L1
L2
i1
i2
Leq = ?
v = Leq d(i1+i2)/dt
Leq
v = L1 di1/dt + M di2/dt + M di1/dt + L2 di2/dt
L1 case
(L1 - M) di1/dt = (L2 - M) di2/dt
di1/dt = (L2 - M)/(L1 - M) di2/dt
v = L1 di1/dt + M di2/dt = L1 (L2 - M)/(L1 - M) di2/dt + M di2/dt
= [L1(L2 - M)/(L1 - M) + M] di2/dt = (L1L2 - L1M + (L1 - M)M)/(L1 - M) di2/dt
(L1L2 - M^2)/(L1 - M) di2/dt


Leq di1/dt + Leq di2/dt = Leq ((L2 - M)/(L1 - M) di2/dt) + Leq di2/dt
= [Leq ((L2 - M)/(L1 - M) + 1)] di2/dt
= L1 [((L2 - M)/(L1 - M)) di2/dt] + M di2/dt
= (L1 (L2 - M)/(L1 - M) + M) di1/dt = [(L1L2 - L1M + (L1 - M)M)/(L1 - M)] di2/dt
Leq ((L2 - M)/(L1 - M) + 1) = (L1L2 - M^2)/(L1 - M)
Thus, Leq = (L1L2 - M^2)/((L1 - M)[(L2 - M)/(L1 - M) + 1]) = (L1L2 - M^2)/(L2 - M + L1 - M) = (L1L2 - M^2)/(L1 + L2 - 2M)
Leq = (L1L2 - M^2)/(L1 + L2 - 2M)


3H = M
(-3H)
L1 = 5H
6H = L2
Leq
= (L1L2 - M^2)/(L1 + L2 - 2M)
= (5 × 6 - 3^2)/(5 + 6 - 2(3))
= 21/5 = 4.2H

phasors (chap 9)



$V = V_m \angle \phi$ (phasor)

$= V_m \cos\phi + j V_m \sin\phi$

real part; imag part

where $j = \sqrt{-1}$

Example

$v(t) = 10\cos(\omega t + 30^\circ)$

$V = 10\angle 30^\circ$ (phasor)

$v_1(t) = 10\sin(\omega t + 30^\circ)$

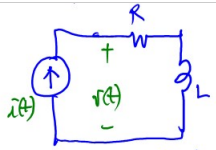
$= 10\cos(\omega t - 90^\circ + 30^\circ) = 10\cos(\omega t - 60^\circ)$

$V_1 = 10\angle -60^\circ$ (phasor)

$\sin\omega t = \cos(\omega t - 90^\circ)$

$= \cos\omega t \cos 90^\circ + \sin\omega t \sin 90^\circ$

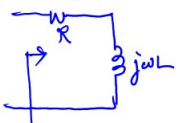
$= \sin\omega t$



$v = Ri + L\frac{di}{dt}$

If $i(t) = e^{j\omega t}$, then $v(t) = R e^{j\omega t} + j\omega L e^{j\omega t}$

$= (R + j\omega L) e^{j\omega t}$



$V(j\omega) = (R + j\omega L)\, I(j\omega)$

$\frac{V(j\omega)}{I(j\omega)} = Z(j\omega) = R + j\omega$

$Z(j\omega) = R + j\omega L = Z\angle\phi_z$ (phasor)

$Z = \sqrt{R^2 + (\omega L)^2}$

$\phi_z = \tan^{-1}\frac{\omega L}{R}$

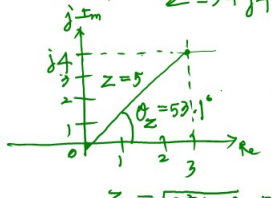
In general, $I = \frac{V}{Z} = \frac{V\angle\theta_v}{Z\angle\theta_z} = \frac{V_m}{Z}\angle\theta_v - \theta_z$

$i(t) = \frac{V_m}{Z}\cos(\omega t + \theta_v - \theta_z)$

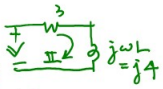
Example

If $R = 3\,\Omega$ and $j\omega L = j4$

$Z = 3 + j4 = 5\angle 53.1^\circ$



$Z = \sqrt{3^2 + 4^2} = 5$



If $R = 3$, $j\omega L = j4$

$Z = 3 + j4 = 5\angle 53.1^\circ$

$\phi_z = \tan^{-1}\frac{4}{3} = 53.1^\circ$