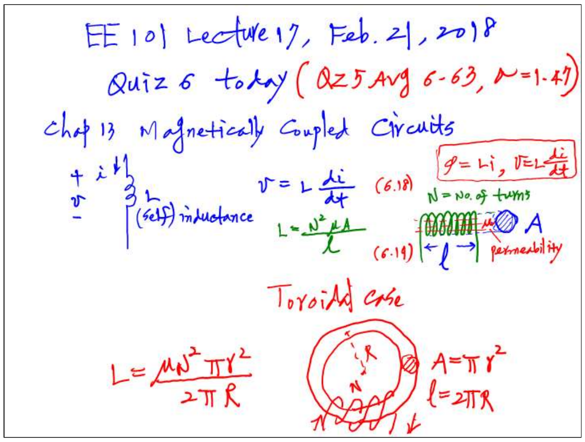
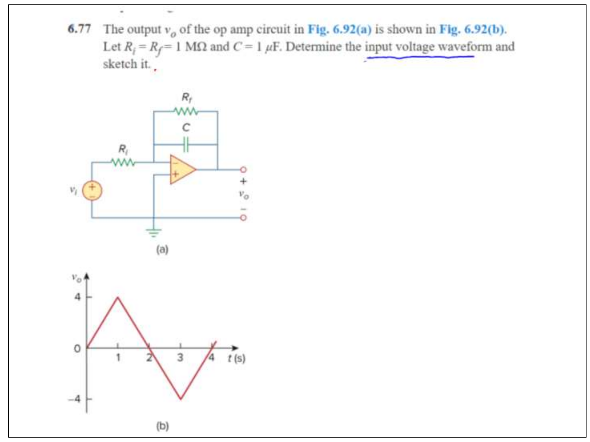
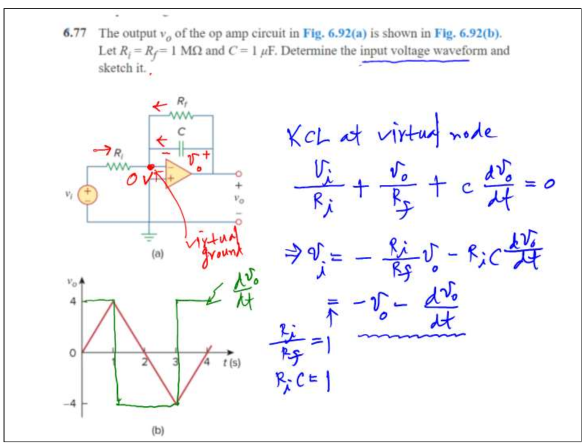
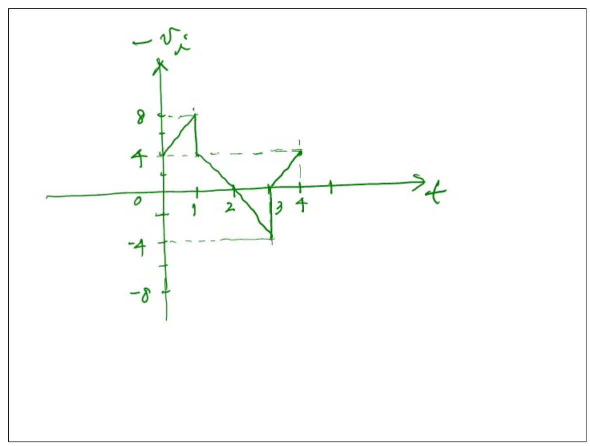
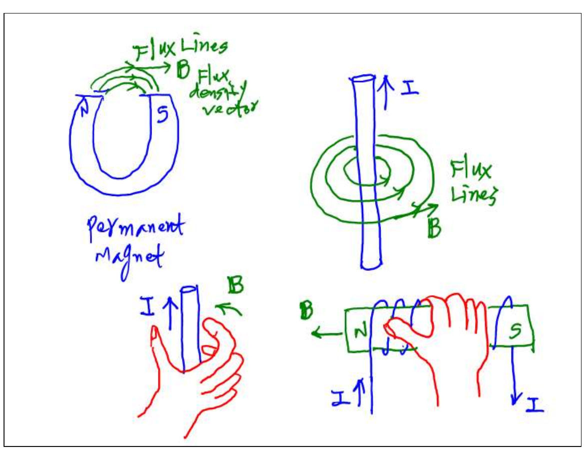
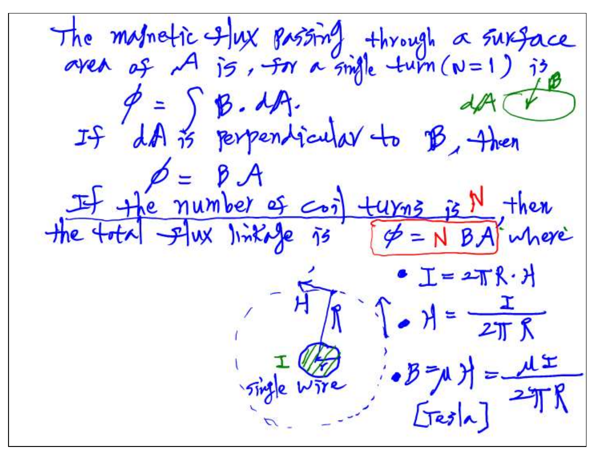

EE 101 Lecture 17, Feb. 21, 2018
Quiz 6 today (QZ5 Avg 6.63, σ = 1.47)
Chap 13 Magnetically Coupled Circuits
φ = Li, v = L di/dt
v = L di/dt (6.18)
(self) inductance
N = No. of turns
L = N²μA / l (6.19)
A
permeability
Toroidal case
L = μN²πr² / 2πR
A = πr²
l = 2πR


6.77 The output v_o of the op amp circuit in Fig. 6.92(a) is shown in Fig. 6.92(b). Let R_i = R_f = 1 MΩ and C = 1 μF. Determine the input voltage waveform and sketch it.
(a)
(b)


6.77 The output v_o of the op amp circuit in Fig. 6.92(a) is shown in Fig. 6.92(b). Let R_i = R_f = 1 MΩ and C = 1 μF. Determine the input voltage waveform and sketch it.
virtual ground
KCL at virtual node
v_i/R_i + v_o/R_f + C dv_o/dt = 0
⇒ v_i = −(R_i/R_f) v_o − R_i C dv_o/dt
= −v_o − dv_o/dt
R_i/R_f = 1
R_i C = 1
(a)
(b)


−v_i
t


Flux Lines
B Flux density vector
N
S
Permanent Magnet
I
Flux Lines
B


The magnetic flux passing through a surface area of A is, for a single turn (N = 1) is
φ = ∫ B · dA
If dA is perpendicular to B, then
φ = BA
If the number of coil turns is N then the total flux linkage is φ = N BA where
• I = 2πR · H
• H = I / 2πR
• B = μH = μI / 2πR [Tesla]
single wire

voltage induced between two ends (with N turns in the body) is

$\phi = NBA$

$v = \frac{d}{dt}(N\phi) = \frac{d}{dt} N(NBA)$

$= \frac{d}{dt} N^2(\mu H)A = \frac{d}{dt}\left[N^2 \mu \frac{I}{2\pi R}\right]\pi r^2$

$= \frac{d}{dt}\left\{N \frac{\mu N}{2\pi R/(\pi r^2)}\right\} \cdot I = \frac{d}{dt} N\left[\frac{NI}{\frac{2\pi R}{\mu \pi r^2}}\right]$ ($=L$)

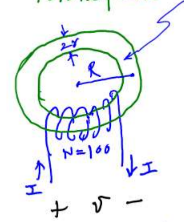
$= \frac{d}{dt} N\left[\frac{NI}{\mathcal{R}}\right]$ where $\mathcal{R} = \frac{2\pi R}{\mu \pi r^2} = \frac{l}{\mu A}$

reluctance

note that $L = \frac{\mu N^2 A}{l}$ where $A = \pi r^2$, $l = 2\pi R$

Example

Toroidal core



$\mu_r = 5000$

$R = 10\,cm$

$r = 2\,cm$

$N = 100$

$\mu = \mu_r \mu_0 = 5000(4\pi \times 10^{-7})$

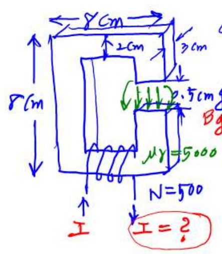
$\phi = \frac{\mu N I r^2}{2R} = \frac{5000(4\pi \times 10^{-7})100 I (2\times 10^{-2})^2}{2\times 10 \times 10^{-2}}$

$= 2.513 \times 10^{-3} I\,[Wb]$

$v = N\frac{d\phi}{dt} = 100 \times 2.513\times 10^{-3}\frac{dI}{dt}$

If $I = 2\sin 200\pi t$ $\quad v = 100 \times 2.513 \times 10^{-3}\frac{d}{dt}(2\sin 200\pi t)$

$= 157.9 \cos 200\pi t\,[V]$



$B_{gap} = 2.5$ Tesla

cross section (2cm × 3cm) $A = 6\,cm^2$

$l = 8\,cm \times 4 - 0.5\,cm = 23.5\,cm$

$\mathcal{R}_{core} = \frac{l}{(\mu_r \mu_0) A} = \frac{23.5\times 10^{-2}\,m}{(5000 \times 4\pi \times 10^{-7})\, 6\times 10^{-4}\,m^2}$

$= 5.195 \times 10^{4}$ A.turns/Wb

$\mathcal{R}_{gap} = \frac{0.5\times 10^{-2}}{\mu_0 A_{fringing}} = \frac{0.5\times 10^{-2}\,m}{4\pi\times 10^{-7}(8.75\times 10^{-4}\,m^2)}$

$= 4.547 \times 10^{6}$ A.turns/Wb

$\phi = B_{gap} A_{gap} = 0.25 \times 8.75\times 10^{-4}$

$= 2.188 \times 10^{-4}\,[Wb]$

$\mathcal{F} = NI$

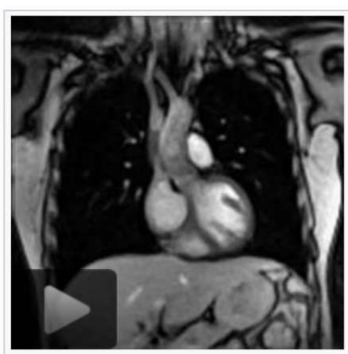
$= \phi(\mathcal{R}_{core} + \mathcal{R}_{gap})$

$= 2.188 \times 10^{-4}\,Wb \cdot (4.6\times 10^{6})\frac{A.turns}{Wb}$

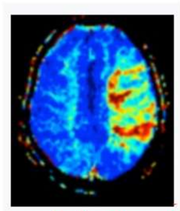
$= 1006$ A.turns

$I = \frac{1006\ A.turns}{500} = 2.012\,A$ (ans)

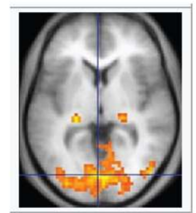
A 3 tesla clinical MRI scanner.



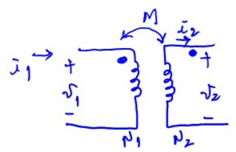
Real-time MRI of a human heart at a resolution of 50 ms milliseconds



cerebral infarction



Localizing highly active brain area before surgery

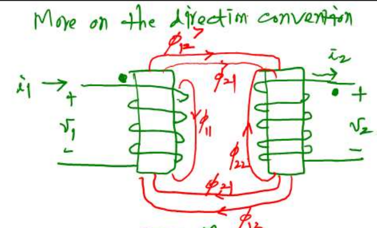


• convention
1) If both currents enters into • terminals, M is +
2) If either current enters in opposite direction, M is −
3) If both currents leaves • terminal, M is +.

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} L_1 & M_{12} \\ M_{21} & L_2 \end{bmatrix} \begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{bmatrix}$$

Here $M_{12} = M_{21} = M$

More on the direction convention

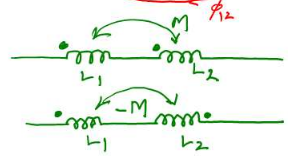


Total flux

$\lambda_1 = \lambda_{11} + \lambda_{12}$

$\lambda_2 = \lambda_{21} + \lambda_{22}$

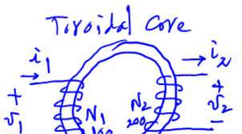
λ_{ij} > 0 when aiding, < 0, opposing

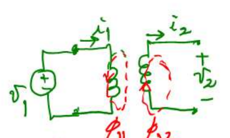


$L = L_1 + L_2 + 2M$

$L = L_1 + L_2 - 2M$

Toroidal Core





Let $\mathcal{R}_{core} = 10^7$ Ampere-turns/Wb λ_{11} λ_{12}

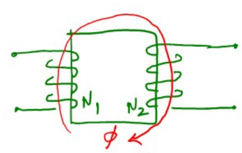
Determine L_1, L_2, M

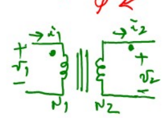
$$L_1 = \frac{N_1^2}{\mathcal{R}} = \frac{(100)^2}{10^7} = 10^{-3} \frac{\text{turns}^2}{\text{A.turns/Wb}} = 10^{-3} \frac{\text{turns}}{\text{A.Wb}} = 1\ mH$$

$$L_2 = \frac{N_2^2}{\mathcal{R}} = \frac{(200)^2}{10^7} = 4\ mH$$

$$\phi_1 = \frac{N_1 i_1}{\mathcal{R}} = \frac{100 i_1}{10^7}, \quad \lambda_{12} = N_2 \phi = 200 \cdot 100 i_1 / 10^7$$

$$M = \lambda_{12} / i_1 = 2\ mH$$





$v_1 = N_1 \frac{d\phi}{dt}$

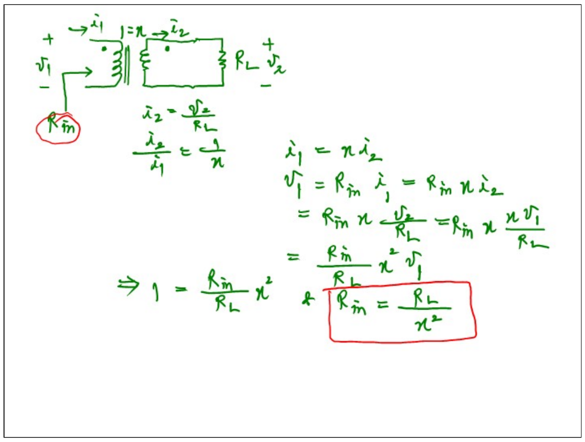
$v_2 = N_2 \frac{d\phi}{dt}$

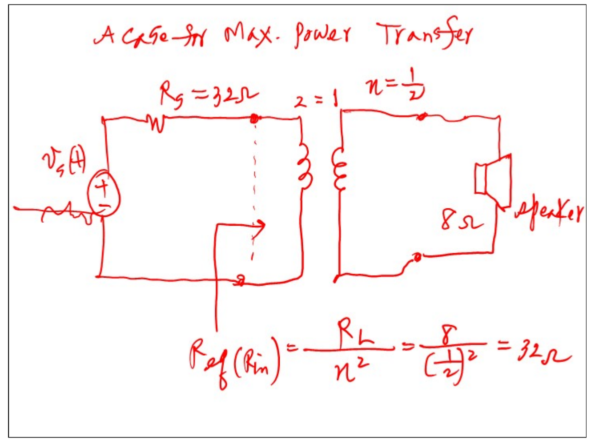
$$\boxed{\frac{v_2}{v_1} = \frac{N_2}{N_1} = n}$$

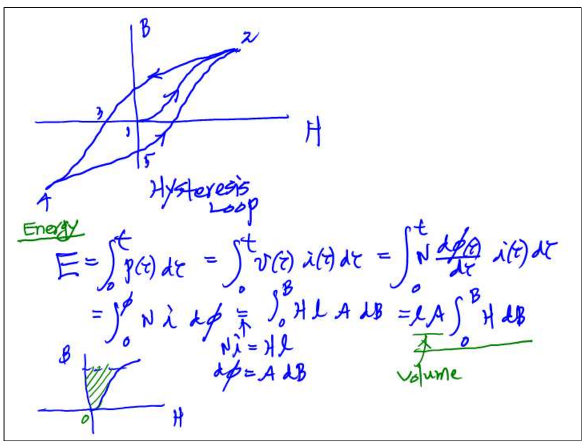
$v_1 i_1 = v_2 i_2 \Rightarrow \frac{v_2}{v_1} = \frac{i_1}{i_2} = \frac{N_2}{N_1}$

(Power conservation)

$$\boxed{\frac{i_2}{i_1} = \frac{N_1}{N_2} = \frac{1}{n}}$$


1 : n
R_L
R_in
i_1 = n i_2
v_1 = R_in i_1 = R_in n i_2
R_in = R_L / n^2


A Case for Max. Power Transfer
R_s = 32Ω
2 : 1
n = 1/2
v_s(t)
8Ω
speaker
R_ef(R_in) = R_L / n^2 = 8 / (1/2)^2 = 32Ω


B
H
Hysteresis Loop
Energy
Ni = Hl
dφ = A dB
Volume