EE101 Lecture 16; Feb. 16, 2018

Qz 6 on Feb 21 (W)

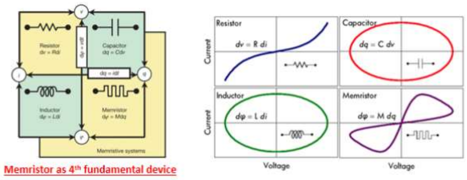


TABLE 6.1

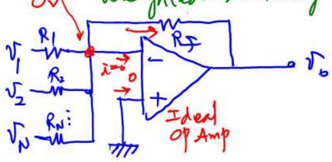
Important characteristics of the basic elements.†

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$ $= G v$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0)$
p or w: E	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

It is appropriate at this point to summarize the most important characteristics of the three basic circuit elements we have studied. The summary is given in Table 6.1.

Quiz 5

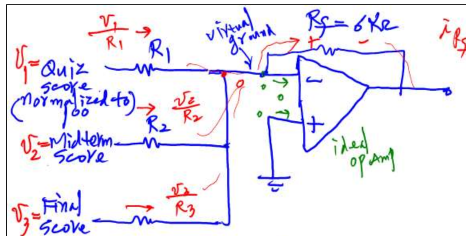
weighted summing circuit



the current through R_f, $i_{R_f} = \sum_{i=1}^{N} \frac{v_i}{R_i}$

Thus, $v_0 = -R_f i_{R_f} = -\sum_{k=1}^{N}\left(\frac{R_f}{R_i}\right)v_i$

$= -\sum_{i=1}^{N} \underbrace{w_i}_{\text{weight}} v_i$



$i_{R_f} = \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}$

v_0 (final grade score) = 20% Qz + 30% mid + 50% Final

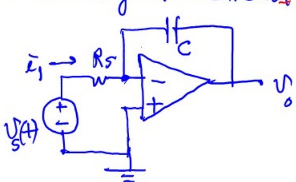
$= 0.2\, v_1 + 0.3\, v_2 + 0.5\, v_3$

$= (-)\left[\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right]$

neglect sign reversal

$\Rightarrow R_1 = R_f/0.2 = 30k\Omega$, $R_2 = R_f/0.3 = 20k\Omega$, $R_3 = R_f/0.5 = 12k\Omega$

An Integrator circuit



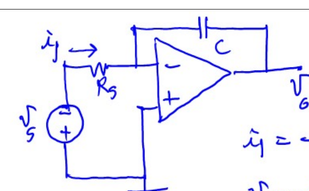
$i_1 = \frac{v_s}{R_s}$

$v_0 = -\frac{1}{C}\int i_1(\tau)\,d\tau$

$= -\frac{1}{C}\int \frac{v_s(\tau)}{R_s}\,d\tau$

$= -\frac{1}{CR_s}\int v_s(\tau)\,d\tau$

when $CR_s = 1$, $\boxed{v_0 = -\int v_s(\tau)\,d\tau}$



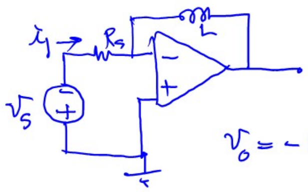
$i_1 = -\frac{v_s}{R_s}$

$v_0 = -\frac{1}{C}\int\left(-\frac{v_s(\tau)}{R_s}\right)d\tau$

$= \int v_s(\tau)\,d\tau$

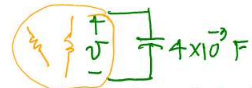
when $CR_s = 1$

A Differentiation Circuit



$$v_o = -L\frac{di_1}{dt}$$
$$= -L\frac{d}{dt}\left(-\frac{v_s}{R_s}\right)$$
$$= \frac{L}{R_s}\frac{dv_s}{dt}$$

when $\frac{L}{R_s} = 1$ $\quad \frac{dv_s}{dt}$



$v(0) = 50\,V$

6.8 A 4-mF capacitor has the terminal voltage

$$v = \begin{cases} 50 \text{ V}, & t \le 0 \\ Ae^{-100t} + Be^{-600t} \text{ V}, & t \ge 0 \end{cases}$$

If the capacitor has an initial current of 2 A, find:

$i(t)\big|_{t=0} = 2A$

(a) the constants A and B,

(b) the energy stored in the capacitor at $t = 0$,

(c) the capacitor current for $t > 0$.

$$i(t) = C\frac{dv(t)}{dt}$$
$$= 4\times10^{-3}[F]\,\frac{d}{dt}\left[\left(Ae^{-100t} + Be^{-600t}\right)[V]\right]$$
$$= 4\times10^{-3}[F]\left[-100Ae^{-100t} - 600Be^{-600t}\right]\left[\frac{V}{s}\right]$$
$$i(0) = 4\times10^{-3}(-100A - 600B) = 2\,[A]$$

Also at $t=0$, $V = 50\,[V] = A + B$

$$\Rightarrow \quad A + B = 50 \qquad \begin{bmatrix} 1 & 1 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 50 \\ -5 \end{bmatrix}$$
$$-0.4A - 2.4B = 2$$
$$A = \frac{\begin{vmatrix} 50 & 1 \\ -5 & 6 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & 6 \end{vmatrix}} = \frac{300-(-5)}{6-1} = \frac{305}{5} = 61$$
$$B = 50 - A = -11$$

Energy stored in the capacitor at $t = 0$

$$E = \frac{1}{2}Cv^2 = \frac{1}{2}\times(4\times10^{-3}\,F)(50\,V)^2$$
$$= 2\times2500\times10^{-3}\,[F\,V\,V]$$
$$= 5\,J$$

$$= \left[\frac{FV}{s}V\,s\right] = [J]$$
$$= [A][V][s]$$
$$[W]$$

$$i(t) = C\frac{dv(t)}{dt} = 4\times10^{-3}\left[-100\times61\,e^{-100t} - 600(-11)e^{-600t}\right]$$
$$= -24.4e^{-100t} + 26.4\,e^{-600t}$$

6.11 A 4-mF capacitor has the current waveform shown in Fig. 6.48. Assuming that $v(0) = 10$ V, sketch the voltage waveform $v(t)$.

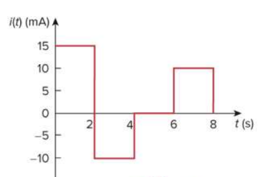
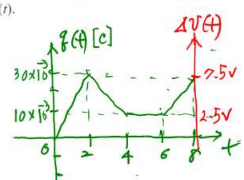
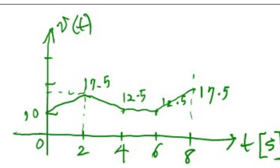


Figure 6.48
For Prob. 6.11.

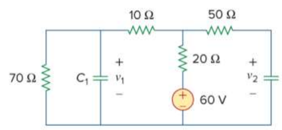


$$v(0) = 10$$
$$v(t) = 10 + \frac{1}{4\times10^{-3}}\int_0^t i(\tau)\,d\tau = 10 + 250\,q(t)$$

$$v(0) = 10$$
$$v(t) = 10 + \frac{1}{C}\int i(\tau)\,d\tau = 10 + \frac{1}{4\times10^{-3}}\int_0^t i(\tau)\,d\tau$$



6.13 Find the voltage across the capacitors in the circuit of **Fig. 6.49** under dc conditions.



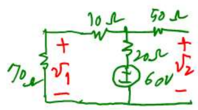


Figure 6.49
For **Prob. 6.13**.

C_1, C_2 are open circuited!

$i_k = C_k \frac{dv_k}{dt} = 0$ for $k = 1, 2$

$v_1 = 60 \frac{70}{70+10+20} = 60 \times 0.7 = 42$ V

$v_2 = 60 - 60 \frac{20}{70+10+20} = 60 - 12 = 48$ V

6.24 In the circuit shown in **Fig. 6.58** assume that the capacitors were initially uncharged and that the current source has been connected to the circuit long enough for all the capacitors to reach steady state (no current flowing through the capacitors). Determine the voltage across each capacitor and the energy stored in each.

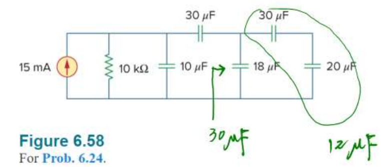
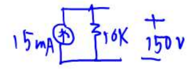
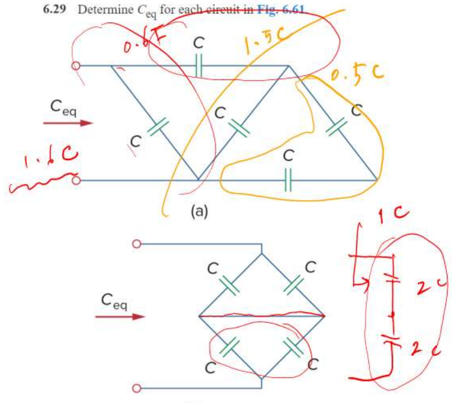
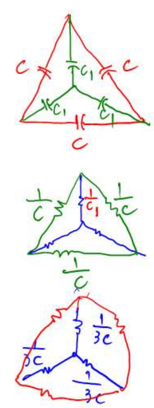


Figure 6.58
For **Prob. 6.24**.



6.29 Determine C_{eq} for each circuit in Fig. 6.61





$v = \left[\frac{1}{C}\right] \int i(t)\, dt$

Treat $\frac{1}{C}$ as R

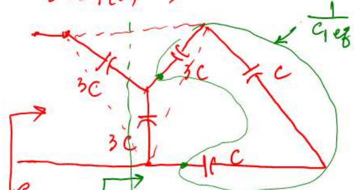
$\frac{1}{C_1} = \frac{\left(\frac{1}{C}\right)\left(\frac{1}{C}\right)}{\frac{1}{C}+\frac{1}{C}+\frac{1}{C}} = \frac{\frac{1}{C^2}}{\frac{3}{C}} = \frac{1}{3C}$

$\boxed{C_1 = 3C}$

$C' = \frac{\left(\frac{1}{3C}\frac{1}{3C}\right) \times 3}{\frac{1}{3C}} = \frac{\frac{1}{3C}\frac{1}{\cancel{3}C}\cancel{3}}{\frac{1}{\cancel{3}C}}$

$= \frac{1}{C}$ ✓ (original value)

6.29(a) ⇒



$\frac{1}{C_{eq}} = \frac{1}{3C} + \frac{1}{C} + \frac{1}{C}$

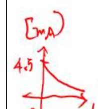
$= \frac{7}{3C}$

$C_{eq} = \frac{3}{7} C$

$C_{eq2} = 3C + \frac{3}{7} C = \frac{24}{7} C$

$C_{eq} = \frac{3C\left(\frac{24}{7}C\right)}{3C + \frac{24}{7}C} = \frac{72C}{45}$

6.32 In the circuit in **Fig. 6.64**, let $i_s = 4.5e^{-2t}$ mA and the voltage across each capacitor is equal to zero at $t = 0$. Determine v_1 and v_2 and the energy stored in each capacitor for all $t > 0$.



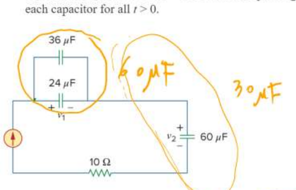
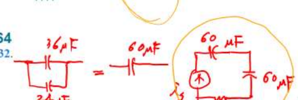


Figure 6.64
For **Prob. 6.32**.



$\int_0^t \frac{dv_2}{dt} dt = A \int_0^t e^{-2t} dt$

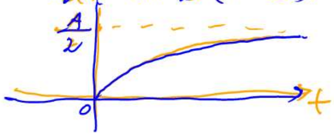
$i_s = C_2 \frac{dv_2}{dt} = 60\mu F \frac{dv_2}{dt} = 4.5e^{-2t} \times 10^{-3}$

$\frac{dv_2}{dt} = \underbrace{\left[\frac{1}{60\mu F} \times 4.5 \times 10^{-3}\right]}_{A} e^{-2t}$

$$\left[\frac{4.5\times10^{-3}}{60\times10^{-6}}\right] = A$$

$$\int_0^t e^{-2\tau}\,d\tau = \frac{e^{-2\tau}}{-2}\Big|_0^t = \frac{1}{2}\left(1-e^{-2t}\right)$$

$v_2(t) = A\frac{1}{2}\left(1-e^{-2t}\right)$ $\quad t=0 \quad v_2(0)=0$



6.53 Find L_{eq} at the terminals of the circuit in **Fig. 6.75**.

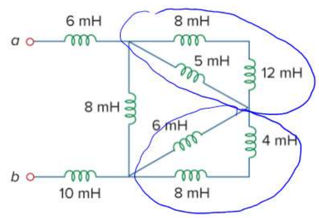


Figure 6.75
For **Prob. 6.53**.

L_{eq} 20 mH

6.57 Determine L_{eq} that may be used to represent the inductive network of **Fig. 6.79** at the terminals.

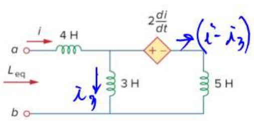


Figure 6.79
For **Prob. 6.57**.

$$\begin{cases} L_{eq}\frac{di}{dt} = 4\frac{di}{dt} + 2\frac{di}{dt} + 5\frac{d(i-i_3)}{dt} \\ 3\frac{di_3}{dt} = 2\frac{di}{dt} + 5\frac{d(i-i_3)}{dt} \\ 8\frac{di_3}{dt} = 7\frac{di}{dt} \end{cases}$$

6.64 The switch in **Fig. 6.86** has been in position A for a long time. At $t = 0$, the switch moves from position A to B. The switch is a make-before-break type so that there is no interruption in the inductor current. Find:

(a) $i(t)$ for $t > 0$,

(b) v *just after* the switch has been moved to position B,

(c) $v(t)$ long after the switch is in position B.

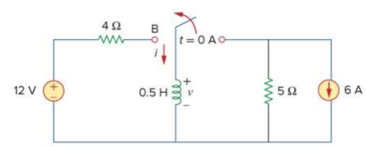


Figure 6.86
For **Prob. 6.64**.

- $i(0) = -6\,A$
- $12 = 4i + 0.5\frac{di}{dt} \Rightarrow 24 = 8i + \frac{di}{dt}$

$\frac{di}{dt} + 8i = 24 \quad i(0) = -6$

$\frac{di}{dt} = 24 - 8i$

$\frac{di}{24-8i} = dt$

$\frac{-\frac{1}{8}d(24-8i)}{24-8i} = dt$

$\frac{d(24-8i)}{24-8i} = -8\,dt$

integration on both sides

$\ln(24-8i)\Big|_{i(0)}^{i(t)} = -8t \rightarrow \frac{24-8i(t)}{24-8(-6)} = e^{-8t}$

$24-8i(t) = 72e^{-8t} \Rightarrow \boxed{i(t) = 3-9e^{-8t}}$

6.77 The output v_o of the op amp circuit in **Fig. 6.92(a)** is shown in **Fig. 6.92(b)**. Let $R_i = R_f = 1\ M\Omega$ and $C = 1\ \mu F$. Determine the input voltage waveform and sketch it.