**The Missing Link in Constitutive Relations**

- Q = CV (Capacitor)
- V = RI (Resistor)
- Φ = LI (Inductor)

\[
\Phi = \int V \, dt
\]

**Capacitor (1745) by**
- Ewald Georg von Kleist
- Resistor (1827) by
- Georg Simon Ohm
- Inductor (1831) by
- Michael Faraday

**Memristor—The missing circuit element**

<table>
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**Abstract**

A new two-terminal circuit element, the memristor, is one of the fundamental components in circuits. Its unique properties make it a versatile device in various applications.

**Memristive devices and systems**

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**Abstract**

A memristor is a device whose resistance changes in response to the amount of charge that has passed through it. This property makes it useful in many applications, including memory storage and analog computing.

**Figure 6.1** A typical capacitor.

**The missing memristor found**

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**Abstract**

Anyone who ever took an electronics laboratory class will be familiar with the fundamental passive circuit elements: the resistor, the capacitor, and the inductor. However, in 1973 Leon Chua reasoned from symmetry arguments that there should be a fourth fundamental element, which he called a memristor (short for memory resistor).
Capacitance $C \ [F]$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

For air, $\varepsilon_0 = 8.85 \times 10^{-12} \text{F/m}$

$\varepsilon_r = 1 - 1.0 \times 10^{-6}$

For air, $\varepsilon = \varepsilon_0 \varepsilon_r = 8.85 \times 10^{-12} \times 1 - 1.0 \times 10^{-6}$

$$C = \varepsilon_0 \frac{1}{L} \text{cm}^2 \text{m} = 8.85 \times 10^{-12} \times 10 = 8.85 \times 10^{-11} \text{F}$$

$$\frac{1}{C} = L \text{m} = \frac{1}{8.85 \times 10^{-11}} \text{m} = 110 \text{mF}$$

$$V(t) = C \frac{d}{dt} \int_0^t i(t) \, dt$$

$$L = \frac{N^2 \mu A}{C}$$

(6.19)

where $N$ is the number of turns, $l$ is the length, $A$ is the cross-sectional area, and $\mu$ is the permeability of the core. We can see from Eq. (6.19) that inductance can be given as

$$L = \frac{N^2 \mu A}{C}$$

Figure 6.21 Typical form of an inductor.
For inductor:

\[ p(t) = v(t) i(t) = L \frac{dv(t)}{dt} \]

\[ E(t) = \int p(t) \, dt = \int L \frac{dv(t)}{dt} \, dt = L \int v(t) \, dv(t) = \frac{1}{2} \left[ i(t)^2 - i(0)^2 \right] \]

Parallel connection:

\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \frac{1}{C_{eq}} = \frac{C_1 C_2}{C_1 + C_2} \]

\[ C_{eq} = C_1 + C_2 \quad C_{eq} = \frac{1}{C_1} + \frac{1}{C_2} \]

\[ C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \quad \frac{1}{C_{eq}} = \frac{1}{3} \quad \frac{1}{C_{eq}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \]

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### TABLE 6.1

<table>
<thead>
<tr>
<th>Relation</th>
<th>Resistor (R)</th>
<th>Capacitor (C)</th>
<th>Inductor (L)</th>
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<tbody>
<tr>
<td>$v/R$</td>
<td>$v = \text{IR}$</td>
<td>$v = \frac{1}{C} \int i(t) , dt + v_i(t)$</td>
<td>$v = L \frac{di}{dt}$</td>
</tr>
<tr>
<td>$i = \frac{v}{R}$</td>
<td>$i = \frac{C}{v} \int i(t) , dt$</td>
<td>$i = \frac{1}{L} \int v(t) , dt + i_0$</td>
<td></td>
</tr>
<tr>
<td>$P = vR$</td>
<td>$P = \frac{v^2}{R}$</td>
<td>$P = \frac{1}{2} C v^2$</td>
<td>$P = \frac{1}{2} L i^2$</td>
</tr>
<tr>
<td>Series</td>
<td>$R_s = R_1 + R_2$</td>
<td>$C_s = C_1 + C_2$</td>
<td>$L_s = L_1 + L_2$</td>
</tr>
<tr>
<td>Parallel</td>
<td>$R_p = \frac{R_1 R_2}{R_1 + R_2}$</td>
<td>$C_p = C_1 + C_2$</td>
<td>$L_p = \frac{L_1 L_2}{L_1 + L_2}$</td>
</tr>
</tbody>
</table>

At dc: Same | Open-circuit | Short-circuit |

*It is appropriate at this point to enumerate the most important characteristics of the three basic circuit elements we have studied. The summary is given in Table 6.1.*
\[ v(t) = \{ 0, t < 0 \} \]
\[ i(t) = \begin{cases} 0, & t \leq 0 \\ \frac{1}{\tau} \int_{0}^{t} v(\tau) \, d\tau + \frac{1}{\tau}, & t > 0 \end{cases} \]
\[ v(t) = \frac{1}{\tau} \int_{0}^{t} v(\tau) \, d\tau \]
\[ P(t) = v(t) i(t) \]
\[ E(t) = \int_{0}^{t} P(t) \, dt = \frac{1}{2} \frac{1}{\tau^2} \]

\[ L = 1 \text{ mH} \]
\[ \dot{i}(t) = L \frac{di(t)}{dt} \]
\[ i(t) = \int_{0}^{t} \frac{1}{L} \, dt = \frac{1}{L} (t + C) \]
\[ v(t) = 10 \sin(\omega t) \]
\[ \sqrt{v(t)} = 10 \frac{\omega}{\sqrt{2}} \sin(\omega t) \]
\[ P(t) = v(t) i(t) = 10 \frac{\omega}{2} \sin(\omega t) \cos(\omega t) \]
\[ E(t) = \int_{0}^{t} P(t) \, dt = 5 \frac{\omega}{2} \sin(\omega t) \cos(\omega t) \]