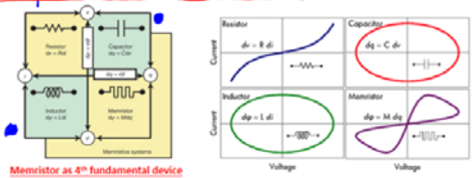
EE101 Lecture 15, Feb. 14, 2018

Qz 5 today

Capacitors & Inductors

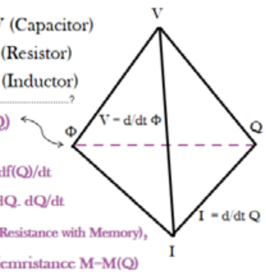



4 Basic Variables

$v, i, q, \varphi$ $\left( \frac{dq}{dt} = i \quad \frac{d\varphi}{dt} = v \right)$

## The Missing Link in Constitutive Relations

- Q = **C** V (Capacitor)
- V = **R** I (Resistor)
- Φ = **L** I (Inductor)
- Φ = f (Q)




dΦ /dt= df(Q)/dt

= df(Q)/dQ. dQ/dt

V= M I (Resistance with Memory),

Thus Memristance M=M(Q)

- **Capacitor (1745)** by Ewald Georg von Kleist
- **Resistor (1827)** by Georg Simon Ohm
- **Inductor (1831)** by Michael Faraday

### Memristor-The missing circuit element




1 Author(s) L. Chua View All Authors


Abstract | Authors | Figures | References | Citations | Keywords | Metrics | Media


**Abstract:**
A new two-terminal circuit element-called the memristorcharacterized by a relationship between the chargeq(t)\equiv \int_{-\infty}^{t} i(\tau) d \tauand the flux-linkage\varphi(t)\equiv \int_{- \infty}^{t} v(\tau) d \tauis introduced as the fourth basic circuit element. An electromagnetic field interpretation of this relationship in terms of a quasi-static expansion of Maxwell's equations is presented. Many circuit-theoretic properties of memistors are derived. It is shown that this element exhibits some peculiar behavior different from that exhibited by resistors, inductors, or capacitors. These properties lead to a number of unique applications which cannot be realized withRLCnetworks alone. Although a physical memristor device without internal power supply has not yet been discovered, operational laboratory models have been built with the help of active circuits. Experimental results are presented to demonstrate the properties and potential applications of memristors.


IEEE Trans. on Circuit Theory, Sept. 1971

### Memristive devices and systems




2 Author(s) L.O. Chua ; Sung Mo Kang View All Authors


Abstract | Authors | Figures | References | Citations | Keywords | Metrics | Media


**Abstract:**
A broad generalization of memristors--a recently postulated circuit element--to an interesting class of nonlinear dynamical systems called memristive systems is introduced. These systems are unconventional in the sense that while they behave like resistive devices, they can be endowed with a rather exotic variety of dynamic characteristics. While possessing memory and exhibiting small-signal inductive or capacitive effects, they are incapable of energy discharge and they introduce no phase shift between the input and output waveforms. This zero-crossing property gives rise to a Lissajous figure which always passes through the origin. Memristive systems are hysteretic in the sense that their Lissajous figures vary with the excitation frequency. At very low frequencies, memristive systems are indistinguishable from nonlinear resistors while at extremely high frequencies, they reduce to linear resistors. These anomalous properties have misled and prevented the identification of many memristive devices and systems-including the thermistor, the Hodgkin-Huxley membrane circuit model, and the discharge tubes. Generic properties


Proc. of the IEEE, Feb. 1976


nature PDF

Letter


### The missing memristor found


Dmitri B. Strukov, Gregory S. Snider, Duncan R. Stewart & R. Stanley Williams



*Nature* **453**, 80–83 (01 May 2008)

doi:10.1038/nature06932

Download Citation

Received: 06 December 2007

Accepted: 17 March 2008

Published online: 01 May 2008

Corrigendum: 25 June 2009


#### Abstract


Anyone who ever took an electronics laboratory class will be familiar with the fundamental passive circuit elements: the resistor, the capacitor and the inductor. However, in 1971 Leon Chua reasoned from symmetry arguments that there should be a fourth fundamental element, which he called a memristor (short for memory resistor)[1].


Capacitor

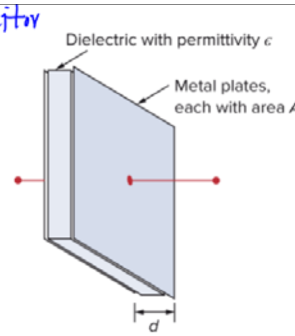
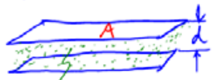



**Figure 6.1** A typical capacitor.

# Capacitance C [F]




$\epsilon$ (dielectric constant)

$\epsilon = \epsilon_r \epsilon_0$

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m

$\epsilon_r = \begin{cases} 3.9 \text{ for } SiO_2 \\ 11.68 \text{ for Si} \\ 7 \sim 8 \text{ for SiN} \end{cases}$

$C = \epsilon \frac{A}{d}$ For air, $\epsilon \approx \epsilon_0 = 8.854 \times 10^{-12}$ F/m

If $d = 1$ mm, $A = 1$ cm², for air

$C = \epsilon_0 \frac{1 \text{ cm}^2}{0.1 \text{ cm}} = 8.854 \times 10^{-14} \times 10 = 8.854 \times 10^{-13}$ F $= 0.8854$ pF


$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau, \quad C\frac{dv}{dt} = i$$

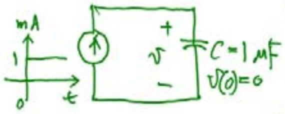
---

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$$

$$= \frac{1}{C}\Big[\underbrace{\int_{-\infty}^{t_0} i(\tau)\,d\tau}_{q(t_0)} + \int_{t_0}^{t} i(\tau)\,d\tau\Big]$$

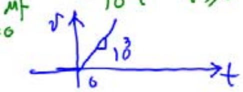
$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau$$

(Example)




$v(t) = 0 + \frac{1}{10^{-6}}\int_0^t (1 \times 10^{-3})\,d\tau$

$= 10^3 t \quad t \geq 0$

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# Inductor

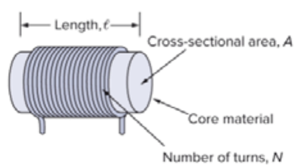


**Figure 6.21** Typical form of an inductor.

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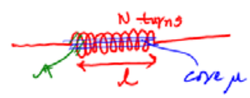


shown in **Fig. 6.21**,

$$L = \frac{N^2 \mu A}{\ell} \qquad \textbf{(6.19)}$$

where $N$ is the number of turns, $\ell$ is the length, A is the cross-sectional area, and $\mu$ is the permeability of the core. We can see from **Eq. (6.19)** that inductance can

$\mu_0 = \mu|_{air} = 4\pi \times 10^{-7}$ henrys/m $\approx 1.257 \times 10^{-6}$ henrys/m

$\mu = \mu_r \mu_0 = 6.3 \times 10^{-3}$ ($\mu_r \approx 5000$) for iron

$2.3 \times 10^{-2}$ ($\mu_r = 18000$) for Cobalt iron

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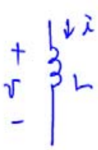



$$v(t) = L\frac{di(t)}{dt}$$

$$\int_{-\infty}^{t} v(\tau)\,d\tau = L[i(t) - \underset{=0}{i(-\infty)}]$$

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\,d\tau$$

$$= \frac{1}{L}\Big[\int_{-\infty}^{t_0} v(\tau)\,d\tau + \int_{t_0}^{t} v(\tau)\,d\tau\Big]$$

$$= \frac{1}{L}\Big[\varphi(t_0) + \int_{t_0}^{t} v(\tau)\,d\tau\Big]$$

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau$$

---

$p(t) = v(t)\, i(t)$

For capacitor,

Power $p(t) = v(t)\, i(t)$

$= v(t)\, C\frac{dv(t)}{dt}$

Energy stored in the capacitor

$$E(t) = \int_{-\infty}^{t} p(\tau)\,d\tau = \int_{-\infty}^{t}\Big[v(\tau)\, C\frac{dv(\tau)}{d\tau}\Big]d\tau$$

$$= C\int_{v(-\infty)}^{v(t)} v\,dv = \frac{1}{2}C\big[v^2(t) - \underset{=0}{v^2(-\infty)}\big] = \frac{1}{2}Cv^2(t)$$

For inductor

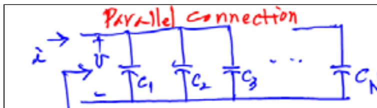
$$p(t) = v(t)\, i(t) = L\frac{di(t)}{dt}\cdot i(t)$$

$$E(t) = \int_{-\infty}^{t} p(\tau)\, d\tau = L\int_{-\infty}^{t} i(\tau)\frac{di(\tau)}{d\tau}\, d\tau = L\int_{i(-\infty)}^{i(t)} i\, di = \frac{1}{2}L\left[i^2(t) - i^2(-\infty)\right]$$

($i^2(-\infty) = 0$)

$$= \frac{1}{2}L\, i^2(t)$$

Parallel connection

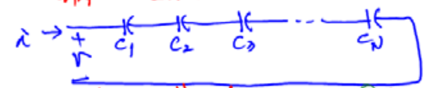



$C_{eq} = ?$

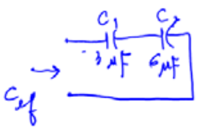
$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt} = (C_1 + C_2 + \cdots + C_N)\frac{dv}{dt}$$

$$C_{eq} = C_1 + C_2 + \cdots + C_N$$

Serial connection




$$\frac{1}{C_{eq}} = \sum_{K=1}^{N}\frac{1}{C_K} \qquad v = \frac{1}{C_{eq}}\int i\, d\tau = \left(\sum \frac{1}{C_K}\right)\int i\, d\tau$$




$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$$

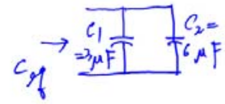
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

similar to

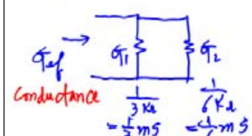

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{3K\Omega \cdot 6K\Omega}{3K\Omega + 6K\Omega} = 2K\Omega$$

$(G_{eq} = G_1 + G_2)$

$$C_{eq} = \frac{3\mu F \cdot 6\mu F}{3\mu F + 6\mu F} = 2\mu F$$




$$C_{eq} = C_1 + C_2 = 3\mu F + 6\mu F = 9\mu F$$




$$G_{eq} = G_1 + G_2 = \frac{1}{3}mS + \frac{1}{6}mS = \frac{2+1}{6}\, mS = \frac{1}{2}mS$$

$$v = \frac{1}{C_1}\int i(\tau)\, d\tau + \frac{1}{C_2}\int i(\tau)\, d\tau + \cdots + \frac{1}{C_N}\int i(\tau)\, d\tau$$

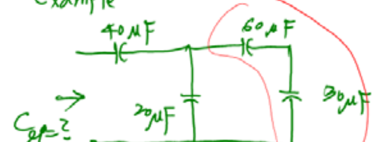
$$= \left[\sum_{K=1}^{N}\frac{1}{C_K}\right]\int i(\tau)\, d\tau = \frac{1}{C_{eq}}\int i(\tau)\, d\tau$$

$$\frac{1}{C_{eq}} = \sum_{K=1}^{N}\frac{1}{C_K}$$

or

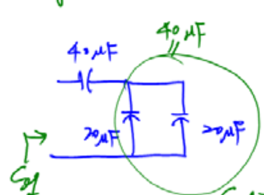
$$C_{eq} = \frac{1}{\sum_{K=1}^{N}\frac{1}{C_K}}$$

Example

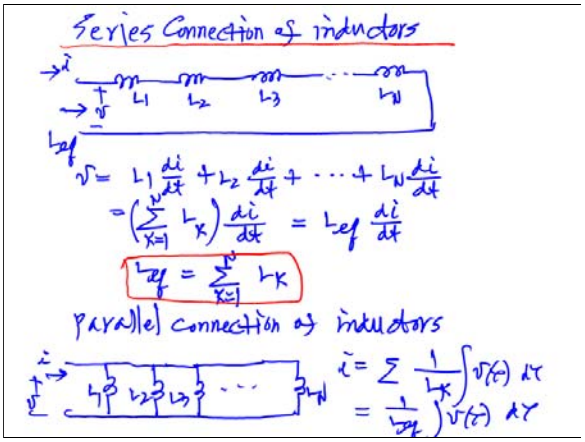


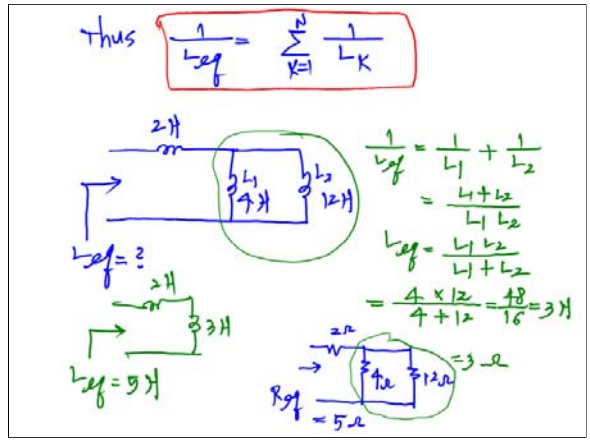

$$\frac{1}{C_{eq1}} = \frac{1}{60\mu F} + \frac{1}{30\mu F}$$

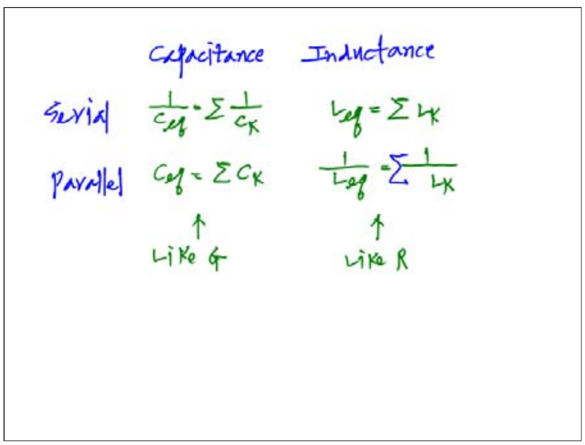
$$C_{eq1} = \frac{1}{\frac{1}{60\mu F} + \frac{1}{30\mu F}} = \frac{60\mu F}{1 + 2} = 20\mu F$$




$$C_{eq} = \frac{1}{\frac{1}{40\mu F} + \frac{1}{40\mu F}} = \frac{40\mu F}{1+1} = 20\mu F$$



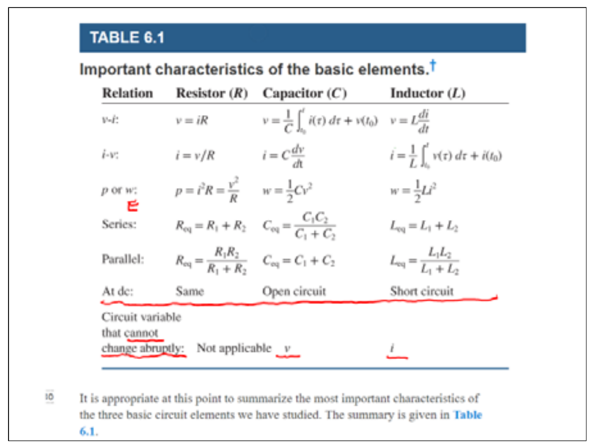
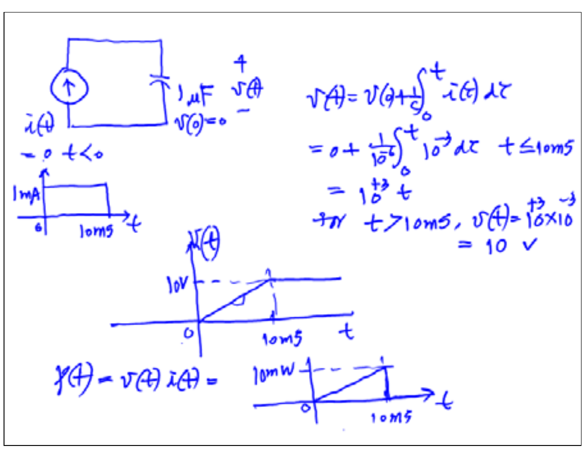



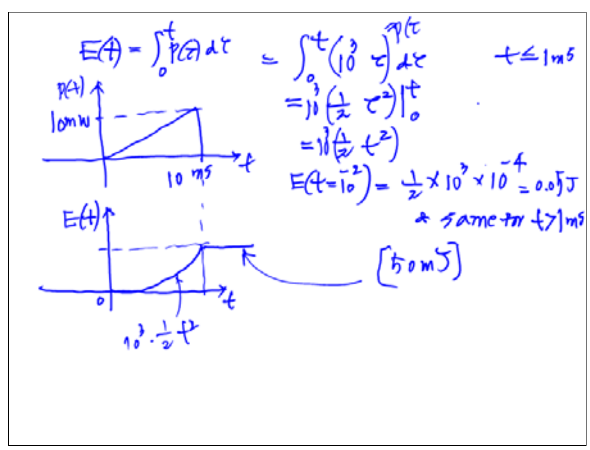
**TABLE 6.1**

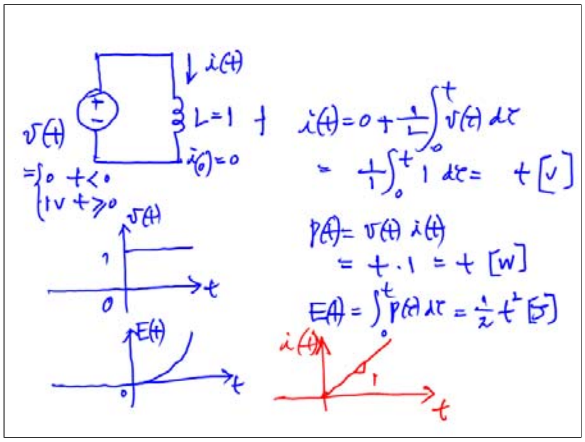
**Important characteristics of the basic elements.**†

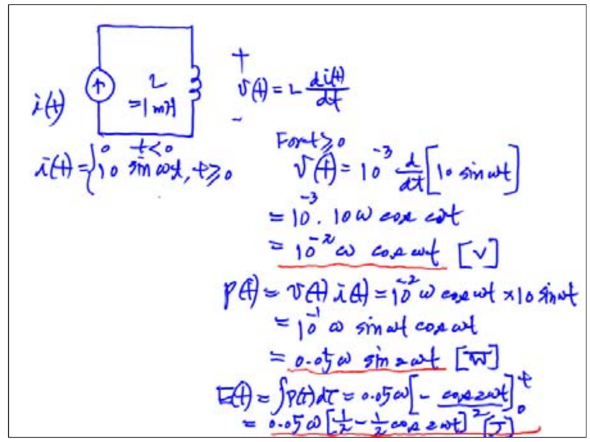
| Relation | Resistor (R) | Capacitor (C) | Inductor (L) |
|---|---|---|---|
| v-i: | $v = iR$ | $v = \frac{1}{C}\int_{t_0}^{t} i(\tau)\, d\tau + v(t_0)$ | $v = L\frac{di}{dt}$ |
| i-v: | $i = v/R$ | $i = C\frac{dv}{dt}$ | $i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\, d\tau + i(t_0)$ |
| p or w: | $p = i^2R = \frac{v^2}{R}$ | $w = \frac{1}{2}Cv^2$ | $w = \frac{1}{2}Li^2$ |
| Series: | $R_{eq} = R_1 + R_2$ | $C_{eq} = \frac{C_1C_2}{C_1 + C_2}$ | $L_{eq} = L_1 + L_2$ |
| Parallel: | $R_{eq} = \frac{R_1R_2}{R_1 + R_2}$ | $C_{eq} = C_1 + C_2$ | $L_{eq} = \frac{L_1L_2}{L_1 + L_2}$ |
| At dc: | Same | Open circuit | Short circuit |
| Circuit variable that cannot change abruptly: | Not applicable | $v$ | $i$ |

10 It is appropriate at this point to summarize the most important characteristics of the three basic circuit elements we have studied. The summary is given in Table 6.1.




i(t)
v(t)
L=1
i(0)=0
= {0 t<0, 1V t≥0
i(t) = 0 + 1/L ∫_0^t v(τ) dτ
= 1/1 ∫_0^t 1 dτ = t [V]
p(t) = v(t) i(t)
= t.1 = t [W]
E(t) = ∫_0^t p(τ) dτ = 1/2 t^2 [J]
v(t)
1
0
t
E(t)
i(t)
1


i(t)
L =1mH
v(t) = L di(t)/dt
i(t) = {0 t<0, 10 sin ωt, t≥0
For t≥0
v(t) = 10^-3 d/dt [10 sin ωt]
= 10^-3 . 10ω cos ωt
= 10^-2 ω cos ωt [V]
p(t) = v(t) i(t) = 10^-2 ω cos ωt × 10 sin ωt
= 10^-1 ω sin ωt cos ωt
= 0.05ω sin 2ωt [W]
E(t) = ∫p(t)dτ = 0.05ω [ - cos 2ωt / 2 ]_0^t
= 0.05ω [1/2 - 1/2 cos 2ωt] [J]