EE101 Final Examination, 4-7 p.m. March 20, 2018

Name_____________________ Student ID______________________

2 pages of formulas and tables only are allowed. But, you must show all your work even when you apply formulas to get full credits. Otherwise, you will lose points.

Prob 1 [10] ___________

Prob 2 [5] ___________

Prob 3 [10] ___________

Prob 4 [10] ___________

Prob 5 [10] ___________

Prob 6 [10] ___________

Prob 7 [10] ___________

Prob 8 [5] ___________

Prob 9 [5] ___________

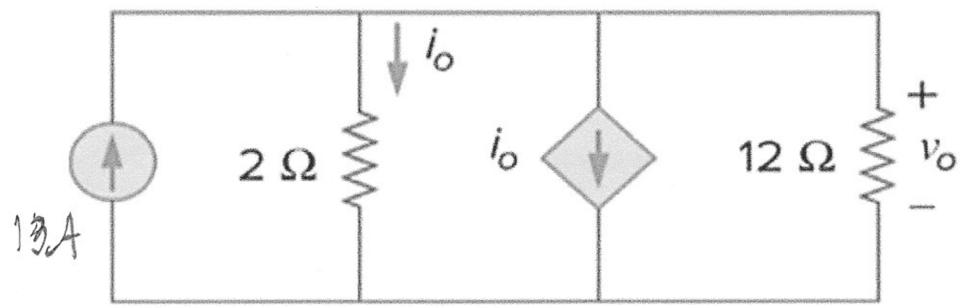
Prob 10[~~10~~ 5] ___________

Prob 11[5] ___________

Prob 12[10] ___________

TOTAL [~~100~~ 95] ___________

[1] (10 points) Find the current i_o and v_o for a current source of 13 A.



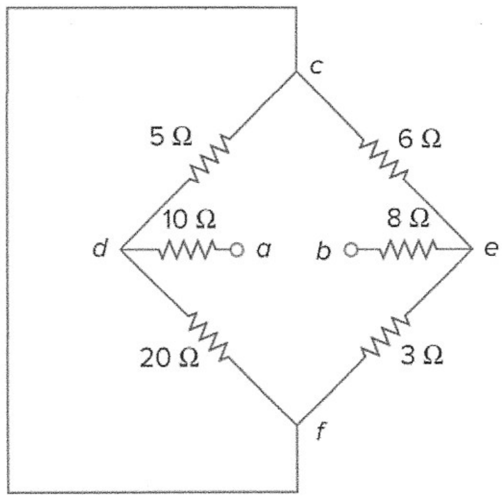
KCL $\quad 13 = \frac{V_0}{2} \times 2 + \frac{V_0}{12}$

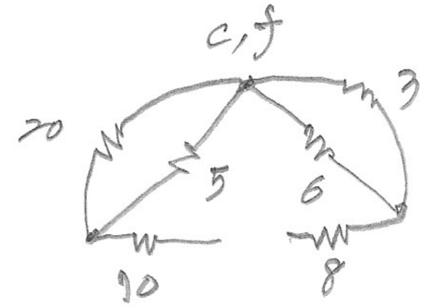
Multiplying both sides by 12 yields

$$13 \times 12 = 12 V_0 + V_0 = 13 V_0$$

$$\boxed{\begin{aligned} V_0 &= 12\,V \\ i_0 &= \frac{12}{2} = 6\,A \end{aligned}}$$

[2] (5 points) Find the equivalent resistance between terminals a and b.





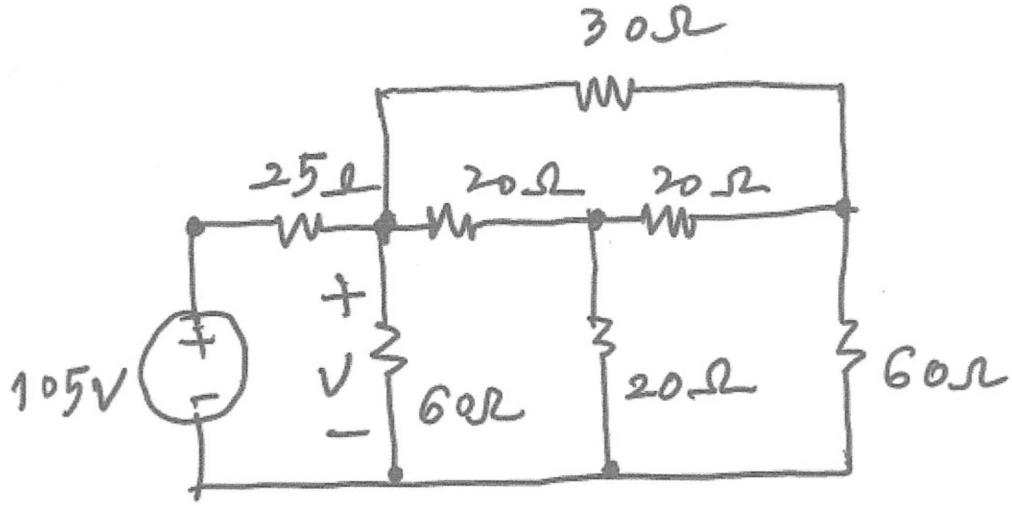
$$20 \| 5 = \frac{100}{25} = 4$$

$$3 \| 6 = \frac{18}{9} = 2$$

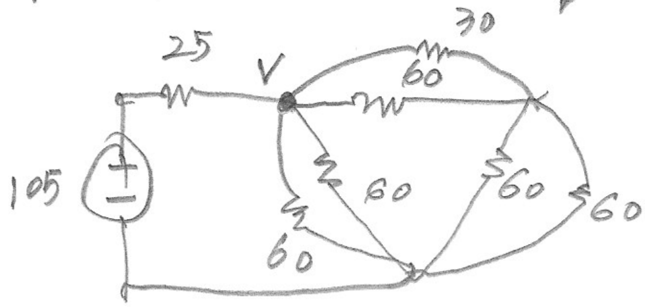
$$R_{ab} = 10 + 4 + 2 + 8$$

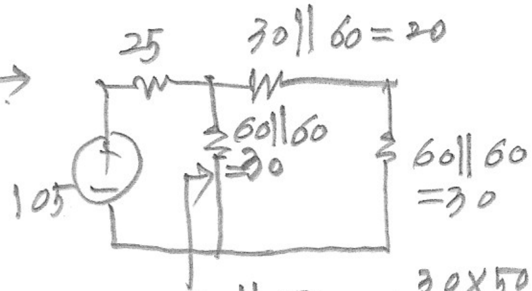
$$= \boxed{24\ \Omega}$$

[3] (10 points) Determine the voltage V in the circuit below.



Y-Δ transformation





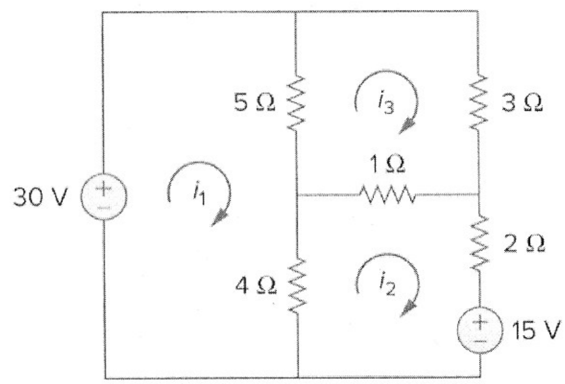
$$30 \| 50 = \frac{30 \times 50}{30 + 50} = \frac{150}{8}\,\Omega$$

$$V = 105 \, \frac{\frac{150}{8}}{25 + \frac{150}{8}}$$

$$= 105 \, \frac{150}{25 \times 8 + 150} = 105 \, \frac{\cancel{150}^{\,3}}{\cancel{350}_{\,7}}$$

$$= \boxed{45\,V}$$

[4] (10 points) Find the **mesh current** i_1.



Mesh equations

Mesh 1 $\quad -30+5(i_1-i_3)+4(i_1-i_2)=0$

2 $\quad 15+4(i_2-i_1)+1(i_2-i_3)+2i_2=0$

3 $\quad 3i_3+1(i_3-i_2)+5(i_3-i_1)=0$

$$\begin{bmatrix} 9 & -4 & -5 \\ -4 & 7 & -1 \\ -5 & -1 & 9 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 30 \\ -15 \\ 0 \end{bmatrix}$$

$$i_1 = \frac{\begin{vmatrix} 30 & -4 & -5 \\ -15 & 7 & -1 \\ 0 & -1 & 9 \end{vmatrix}}{\begin{vmatrix} 9 & -4 & -5 \\ -4 & 7 & -1 \\ -5 & -1 & +9 \end{vmatrix}} = \frac{1245}{199} \simeq \underline{6.26 \text{ amp.}}$$

ans

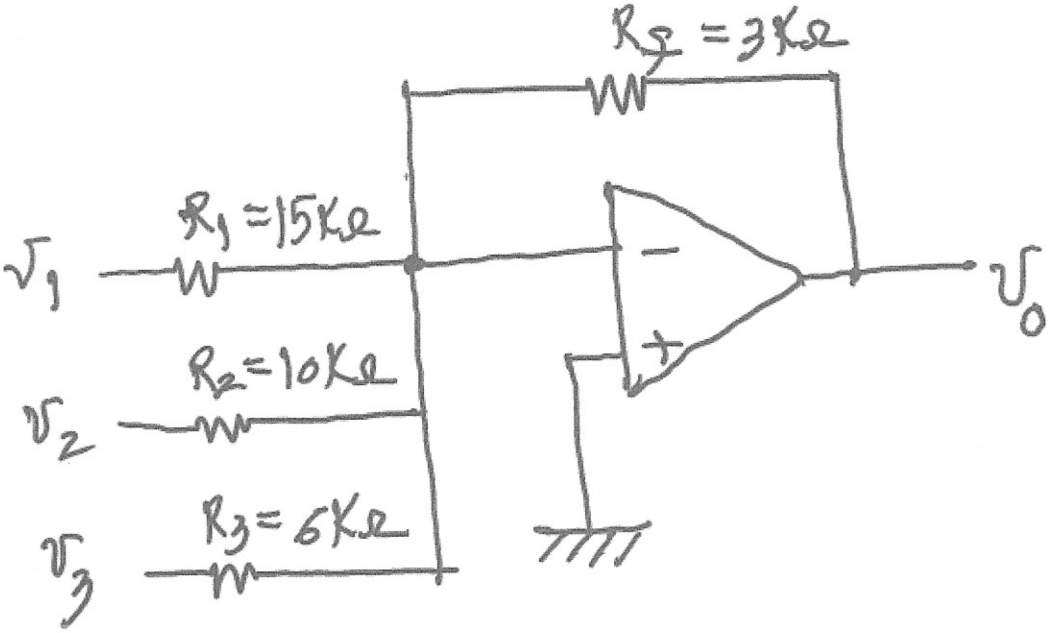
denominator: $9\times7\times9+(-1)(-4)(-5)+(-5)(-4)(-1)$
$-(-5)7(-5)-(-4)(-4)(9)-9(-1)(-1)=199$

numerator $=30(7)(9)+(-15)(-1)(-5)+0-0-(-15)(-4)(9)-30(-1)(-1)$
$=1245$

(for continuation of problem 4 solution)

[5] (10 points) (a). (8 points) Find an expression for V_o in terms of V_1, V_2, and V_3.

(b). (2 points) What is the value of V_o when (V1, V2, V3) = (75, 80, 90)?



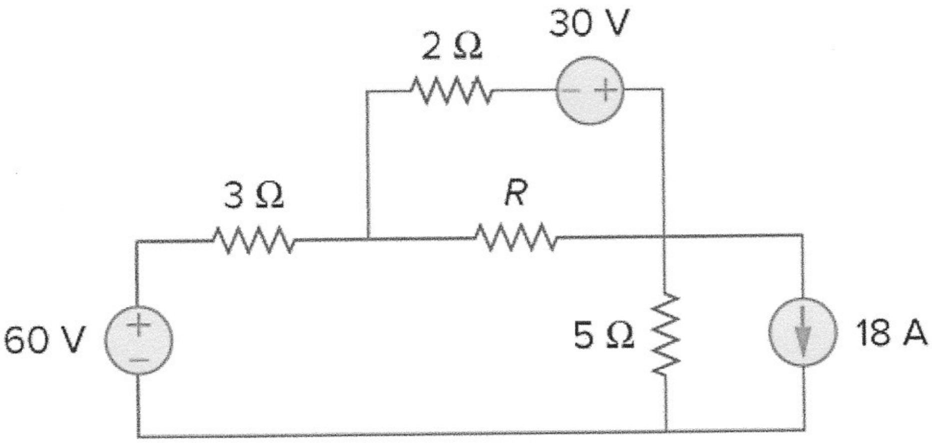
a) $v_o = -\frac{3}{15} v_1 - \frac{3}{10} v_2 - \frac{3}{6} v_3$

$= \underline{-(0.2 v_1 + 0.3 v_2 + 0.5 v_3)}$

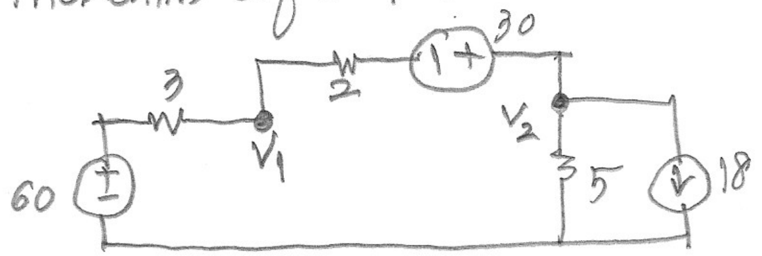
b) $v_o = -(0.2 \times 75 + 0.3 \times 80 + 0.5 \times 90)$

$= -(15 + 24 + 45) = \underline{-84}$

[6] (10 points) Find the maximum power that can be transferred to resistor R in the circuit below and find its maximum power dissipation.



Thevenin's equivalent circuit across R :



KCL at V_1 node : $\dfrac{60-V_1}{3} = \dfrac{V_1+30-V_2}{2} \xrightarrow{\times 6} 120-2V_1 = 3V_1+90-3V_2$

$\boxed{5V_1 - 3V_2 = 30}$ (1)

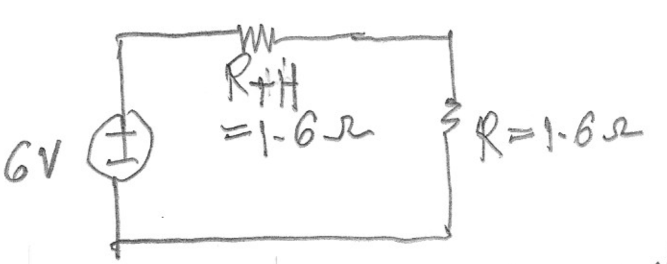
" V_2 " : $\dfrac{V_1+30-V_2}{2} = \dfrac{V_2}{5} + 18 \xrightarrow{\times 10} 5V_1+150-5V_2 = 2V_2+180$

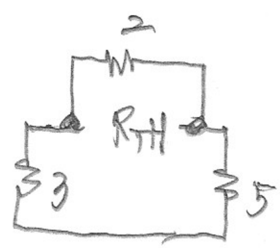
$\boxed{5V_1 - 7V_2 = 30}$ (2)

(1) − (2) → $4V_2 = 0$ $\boxed{V_2 = 0}$

From (1) $5V_1 - 3(0) = 30$ $\boxed{V_1 = 6}$

$\Big\}$ $V_{oc} = 6 - 0 = \underline{6\,V}$





$R_{TH} = 2 \| (3+5)$

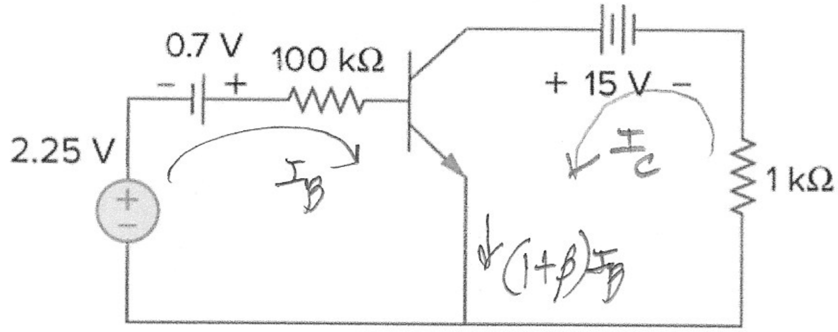
$= \dfrac{2 \times 8}{2+8}$

$= \underline{1.6\ \Omega}$

$V_R = 3V$ $P_R = \dfrac{V_R^2}{R} = \dfrac{3^2}{1.6} = \dfrac{9}{1.6} = \boxed{5.6\ W}$

ans

[7] (10 points) Find the voltage across the collector and emitter terminals of the bipolar junction transistor (BJT) below with β=100 and V_{BE}=0.7 V for the circuit below.



Loop (Mesh) eq.

$$-2.25 - 0.7 + (100K) I_B + 0.7 = 0$$

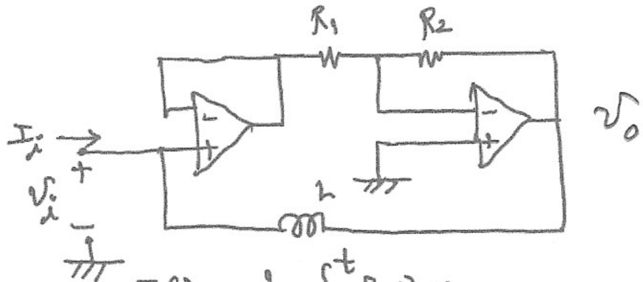
$$(100K) I_B = 2.25 \Rightarrow I_B = \frac{2.25}{100K} = 22.5 \, \mu A$$

$$I_c = \beta I_B = 100 \, I_B = 2.25 \, mA$$

$$V_{CE} = 15 - (1 \, K\Omega) \, 2.25 \, mA = \underline{12.75 \, V}$$

ans

[8] (5 points) Find an expression for L_{eq} in terms of R_1 , R_2 and L.



$$I_i(t) = \frac{1}{L_{eq}} \int_{-\infty}^{t} v_i(\tau)\, d\tau$$

$$I_i = \frac{1}{L} \int_{-\infty}^{\infty} (v_i - v_o)\, d\tau \qquad (1)$$

since $v_o = -\frac{v_i}{R_1} R_2 = -\frac{R_2}{R_1} v_i$

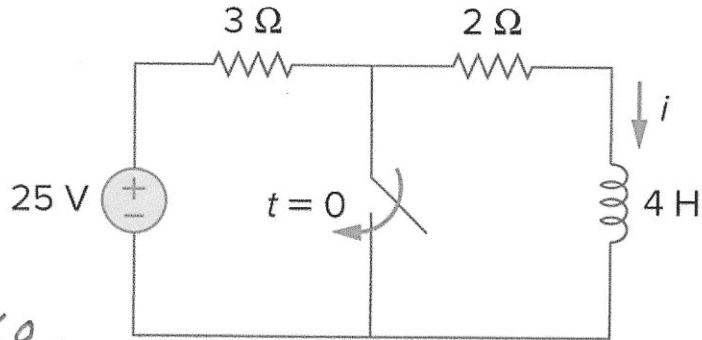
$$(1) \rightarrow \quad I_i = \frac{1}{L} \int_{-\infty}^{t} \left[v_i - \left(-\frac{R_2}{R_1}\right) v_i \right] d\tau$$

$$= \frac{1}{L} \int_{-\infty}^{t} \left[1 + \frac{R_2}{R_1}\right] v_i \, d\tau$$

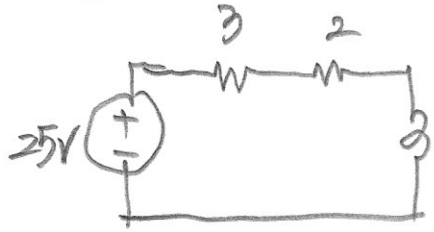
$$= \frac{1}{\left(\frac{L}{1 + \frac{R_2}{R_1}}\right)} \int_{-\infty}^{t} v_i \, d\tau$$

thus $$L_{eq} = \frac{L}{1 + \frac{R_2}{R_1}}$$ ⊠

[10] (5 points) Find the inductor current i(t) and $\frac{di(t)}{dt}$ for both $t < 0$ and $t > 0$.



For $t<0$,



$i(0_-) = \frac{25}{5} = \underline{5\,A}$

For $t>0$

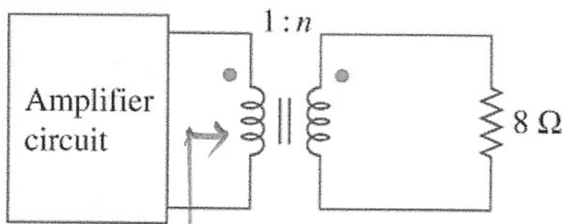
$i(0_+) = i(0_-) = 5$

$L\frac{di(0_+)}{dt} + i(0_+)\cdot 2 = 0$

$\Rightarrow \frac{di(0_+)}{dt} = \frac{-5\times 2}{4}$

$= \underline{-2.5\ [A/s]}$

[11] (5 points) Determine the transformer ratio n for maximum power transfer to the 8 Ω load when the Thevenin's equivalent voltage and the equivalent resistance for the amplifier circuit are 10V and 128 Ω. Assume that the transformer is an ideal transformer ~~and there is zero mutual coupling, i.e. M=0.~~



$$R_{eq} = \frac{8}{n^2}$$

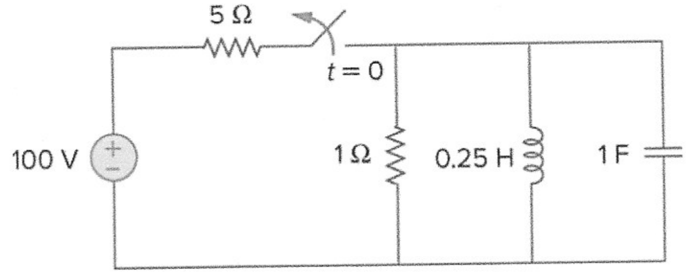
For max power transfer

$$128 = R_{eq} = \frac{8}{n^2}$$

$$n^2 = \frac{8}{128}$$

$$n = \sqrt{\frac{8}{128}} = \frac{1}{16} = \frac{1}{4}$$

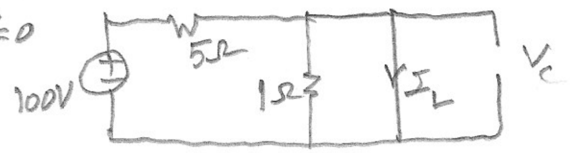
[12] (10 points) Find the voltage across 1F capacitor v(t) for t > 0.



$\omega_d = \sqrt{(\omega_0^2 - \alpha^2)} = \frac{\sqrt{15}}{2}$

$\omega_0 > \alpha \Rightarrow$ under damped

For $t \le 0$



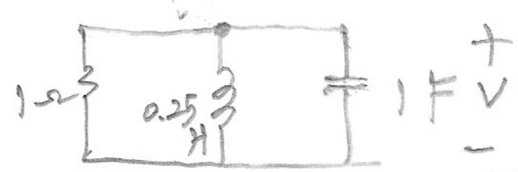
$I_L(0) = 20\,A$

$V_c(0) = 0\,V$

$\boxed{\alpha = \frac{1}{2RC} = \frac{1}{2}}$

$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} = 2}$

For $t > 0$



KCL $\rightarrow \frac{v}{1} + \frac{1}{0.25}\int_0^t v(\tau)\,d\tau + \frac{dv}{dt} = 0$

$\frac{d}{dt} \rightarrow \frac{dv}{dt} + 4v + \frac{d^2v}{dt^2} = 0$ characteristic eq =

$s^2 + 1s + 4 = 0$

$\alpha = 0.5$ $\omega_0^2 = 4$ $(\omega_0 = 2)$

$\omega_0 > \alpha$ underdamped.

$v(t) = M e^{-0.5t}\cos(\omega_d t - \theta)$, where $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{3.75}$

$v(0) = M\cos(-\theta) = 0 \Rightarrow \theta = 90^\circ$

$v'(0) = -0.5\underbrace{M\cos(-\theta)}_{=0} + M(-\omega_d)\sin(-\theta) = -20$

$\frac{dv}{dt} = -i_L$ at $t=0$

$M\sin(-\theta) = \frac{20}{\omega_d} \Rightarrow M = -\frac{20}{\omega_d}$

$v(t) = -\frac{20}{\omega_d} e^{-0.5t}\underbrace{\cos(\omega_d t - 90^\circ)}_{=\sin\omega_d t}$

$\boxed{v(t) = -\frac{20}{\omega_d} e^{-0.5t}\sin\omega_d t}$

at $t=0$, $v(0) = 0$

$v'(0) = \frac{10}{\omega_d}(0) - \frac{20}{\omega_d}(\cos\omega_d t)(\omega_d) = -20$

(Prob. 12 solution continued)